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Finite Different Method and Differential Quadrature Method for Solving Burgers Equation

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ABSTRACT

Finite Difference Method, FDM and Difference Quadrature Method, DQM are the main method used. In order to solve this method in terms of accuracy, the different number nodes and different initial condition are used [1]. The solutions of these methods are compared in terms of accuracy of the numerical solution by using the graph. C++ are used to find numerical solution for those method and the exact solution solve by using maple. For results and tabulate, compare the solution in terms of the accuracy of the numerical solution with the exact solution with the collected result. To find the best method to solve this equation, those method compare by using sum of square error, SSE. The lesser the number of node will affect the increasing the errors of the solution. Generally, in terms of accuracy of numerical solutions, FDM and DQM showed DQM is better than FDM.

Key words : Burgers equation, Differential Quadrature Method, Finite Differential Method.

1. INTRODUCTION

The initial and boundary condition are set for calculating the solution of Burgers' equation within the group theory methods. The application of a one-parameter assembly reduces the number of unconstrained variables by one. The consequently the governing biased differential equation with the boundary state and initial condition to an ordinary differential equilibrium with the appropriate corresponding conditions[2]. The procure differential equilibrium is solution analytically and the solution obtained in closed formality, for a specific choice of boundary condition. Burgers equation loos often as a simplification of a more complicate and debase example. Hence it is usually thought as a folly model, namely, a tool that is used to learn some of the viscera behaviour of the general question. In order to solve the problem, several methods were used to solve the Burgers' Equation. The method used is Finite Different Method and Differential Quadrate Method to solve the Burgers Equation. The Finite Different Method is used for solving PDEs of the Burgers' equation. In this paper, several numerical methods of recount the solutions of initial-boundary condition problems for Burgers' equations in two space dimensions.

The Differential Quadrature Method, which is a honest expansion of its analogy in one space measurement, is very efficient compare with to the other methods, namely the method of lines [3] and a Runge-Kutta-typify [4] regularity as studied by Crouzeix. There is no numeral approximation to obtain the crisis for this nonlinear PDE accordingly too many researchers have adopted different numerical techniques in approach the breach of the Burgers-Huxley equality. However, have been solved the take disintegration for the Burgers-Huxley equality [5]. In this paper, the equation using initial condition, boundary condition and several parameters for solving the Burgers equation [6]. The characteristics of the finite difference method are this rule utilizes uniformly roam grids. Other specific is at each swelling, each secondary is round by an algebraic statement which allusion the adjoining nodes. The solution of the partial differential equation or ordinary differential equation can be approximate by substituting the derivatives expression with the finite difference method. This process is very quiet to explain but it is less particular acquire to DQM [7]. Another of two characteristics are a system of algebraic equations is procure by rate the prior gait for each swelling and the system is explanation for the retainer versatile. The finite difference method (FDM) is usefulness to round the compound of a cosecant import at one moment by worn the sine appraise at disjunctive characteristic. The Differential Quadrature Method is habit to translate the biased discriminating equilibrium to an normal special equilibrium by refund the derivate of the glossy province with a moment narrow alliance importance [8]. This process are utility the performance utility on all grid item, so this system are composite and take the far-reaching knot to find the explanation.

1.1 Methodology

There are seven (7) step involved in this research

Step 1: This step discusses on the general equation of Burgers equation and the corresponding initial boundary conditions.

where x represent length of rod with interval 0 to L, t represent time with interval 0 to τ. The value of L and τ depends on application in engineering field.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = v\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, 0 < t < \tau$$

$$u(0,t) = \psi(x), \qquad 0 < x < L$$

$$u(0,t) = \zeta_1(1), \quad u(L,t) = \zeta_2(t), \quad 0 < t < \tau$$

Step 2: This step discusses about the study of the methods that use to solve the Burgers equation and can be solved using analytically and numerically. For this paper numerical method used to solve the Burgers equation. FDM and DQM are two method used in this project.

Step 3: This step discusses about the determination of the parameter value which is the parameter, initial condition and boundary condition will be setup for FDM and DQM.

Step 4: This step will apply maple software to find the exact solution of the Burgers equation and the solution will tabulated.

Step 5: This step, C++ program were code to find the solution for the Burgers equation for those method.

Step 6: In this step, all the solution that found for FDM and DQM for the Burgers equation compared to the exact solution. The solution compared by finding the error for those two method used.

Step 7: This step will determine the best method found using the accuracy test.

2. IMPLEMENTATION

2.1 Theory of Finite Differential Method

The basic judgment of the method of lines is to refund the boundary value derivative in the partial distinct equations with algebraic approximations. When this is done, the boundary worth derivatives are no longer stated clearly in term of the boundary excellence independent fickle. Therefore, the system of ordinary differential equation has only one remaining self-directing variable in approximation of the inventive partial different equation. Then, the system can apply any integration algorithm for initial value commonplace discriminating equation to compute the numbers numeral breach of biased differential equation.

2.2 Theory of Differential Quadrature Method

A few parameters in the equation such as α, β, ε and γ will be test in custom to compare the sign of different amount of nodes in expression of truth solution. There are two cases contemplate in this meditation will be debate

$$\frac{\partial u}{\partial t} - \varepsilon\frac{\partial^2 u}{\partial x} + \alpha u\left(\frac{\partial u}{\partial x}\right) = \beta(1-u)(u-\gamma)$$

$$a < x < b, t > 0$$

Where
boundary condition
$u(0,t) = 0$ and $u(1,t) = 0$
and initial condition
$u(x,0) = \sin(2\pi x)$

Two case has been used in this study:

Case I

In this case the value of initial condition, the boundary set up as above and the value of $\alpha = 1, \beta = 0, \gamma = 0$. But, the value of parameter, ε set for $\varepsilon = 0.2$, $\varepsilon = 0.4$, $\varepsilon = 0.6$, $\varepsilon = 0.8$ and $\varepsilon = 1.0$. In this project, the value of parameter are setup randomly because the objective of this project to find the best method for solving the Burger's equation but when Burger's equation applied to certain engineering field, the value of parameter setup depend on the boundary value problem in the application field.

$$\frac{\partial u}{\partial t} - \varepsilon\frac{\partial^2 u}{\partial x} + u\left(\frac{\partial u}{\partial x}\right) = 0$$

Case II

In this case the value of initial condition, the boundary set up as above and the value of $\varepsilon = 1, \beta = 0, \gamma = 0$. But, the value of parameter, α set for $\alpha = 0.0$, $\alpha = 0.5$, $\alpha = 1.0$, $\alpha = 1.5$ and $\alpha = 2.0$

$$\frac{\partial u}{\partial t} - \varepsilon\frac{\partial^2 u}{\partial x} + \alpha u\left(\frac{\partial u}{\partial x}\right) = 0$$

3. RESULT AND DISCUSSION

3.1 Introduction

The researcher fixed that the Differential Quadrature Method is an expansion of finite difference method to the maximum custom[9]. In this paper, consider three cases of the Burgers equation. These problems will be solved by using the Differential Quadrature Method and by using the Finite Difference Method in order to compare the solutions between them with the exact solution which is compute by using the Maple software.

Case 1

The equation of the linear Burgers equation is given by:
$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0 \quad 0 < x < 1, \ t > 0$$
with initial condition
$$u(x,0) = x \quad 0 < x < 1$$
and the boundary equations
$$u(0,t) = 0, u(1,t) = 0 \quad t > 0$$

Case II

The equation of the nonlinear Burgers equation is given by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = e^{-1}(1 + x^2) \quad 0 < x < 1,\ t > 0$$

with the initial condition

$$u(x,0) = 1 + x^2 \quad 0 < x < 1$$

and the boundary equations

$$u(0,t) = e^{-1}, \quad u(1,t) = 2e^{-1}, \quad t > 0$$

3.2 Result

Case 1

Two cases had set up and case setup will computed by using Different Quadrature Method in order to get the numerical solution in each case. The exact solution computed by Maple software is used to compare with the numerical solution. In these case we consider the number of nodes, $N = 20, 30$ and 40 with time, $t = 0.03, 0.04, 0.05$. In order to calculate the error of each node, then the formula of error applied.

Error = | exact solution – numerical solution |

Table 1: Exact solution

t/x	0	0.2	0.4	0.6	0.8	1.0
0.03	0	0.2677	0.1718	-0.1762	-0.2681	0
0.04	0	0.1173	0.0813	-0.0832	-0.1173	0
0.05	0	0.0052	0.0134	-0.0136	-0.0052	0

Table 2: The error for $N = 20$ of $u(x)$

t/x	0	0.2	0.4	0.6	0.8	1.0
0.03	0	0.0025	0.0050	0.0006	0.0021	0
0.04	0	0.0015	0.0030	0.0011	0.0014	0
0.05	0	0.0001	0.0006	0.0004	0.0001	0

Case II

Table 3: Exact solution

t/x	0	0.2	0.4	0.6	0.8	1.0
0.03	0	0.5307	0.3681	-0.3681	-0.5307	0
0.04	0	0.2268	0.1845	-0.1845	-0.2268	0
0.05	0	-0.0004	0.0437	-0.0437	0.0004	0

Table 4: The errors for $N = 20$ of $u(x)$

t/x	0	0.2	0.4	0.6	0.8	1
0.03	0	0.0045	0.0111	0.0061	0.0042	0
0.04	0	0.0016	0.0059	0.0035	0.0013	0
0.05	0	0.0008	0.0022	0.0018	0.0007	0

From the tables above, it show the errors for the $N = 20$ which is more stable other than the errors for $N = 30$ and $N = 40$. Solution for DQM will give unstable solution. Therefore, the scalar of nodes in calculating the Burgers equation by using Differential Quadrature Method explains the outcome in name of immovability solution.

The numerical solutions are generated for both FDM and DQM in order to identify the results from both methods in term of accuracy compared to the exact solutions. As discussed in Chapter 3, the solutions with the number of nodes $N = 20$ is better than $N = 10$. Therefore, for this problem, we used $N = 20$ in order to get the best result for both methods. In the table below, the data results generated from FDM, DQM and also the exact value for each time, $t = 0.02$, $t = 0.04$, $t = 0.06$ and $t = 0.08$. FDM will be using C++ software while DQM will be using Maple software.

Case I

Table 5: Results for $t = 0.02$

u(x)\x	0	0.2	0.4	0.6	0.8	1
FDM	0	0.1960	0.3921	0.5882	0.7839	1.0000
DQM	0	0.1972	0.3906	0.5732	0.6888	0.4786
EXACT	0	0.1963	0.3910	0.5742	0.6924	0.4800
ERRORS FDM	0	0.0003	0.0010	0.0140	0.0915	0.52
ERRORS DQM	0	0.0009	0.0004	0.001	0.0035	0.0013

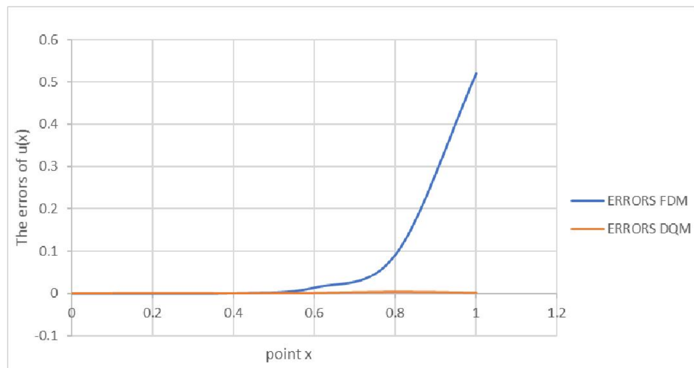

Figure 1: Graph of errors FDM and DQM against $t = 0.02$

Case II

Table 7: Results for $t = 0.003$

u(x)\x	0	0.2	0.4	0.6	0.8	1
FDM	0	0.9999	1.0402	1.1599	1.3595	1.6388
DQM	0	0.9442	1.0355	1.1649	1.3623	1.6247
EXACT	0	0.9442	1.0346	1.1636	1.3618	1.6253
ERRORS FDM	0	0.0557	0.0056	0.0037	0.0023	0.0135
ERRORS DQM	0	0	0.0009	0.0012	0.0004	0.0005

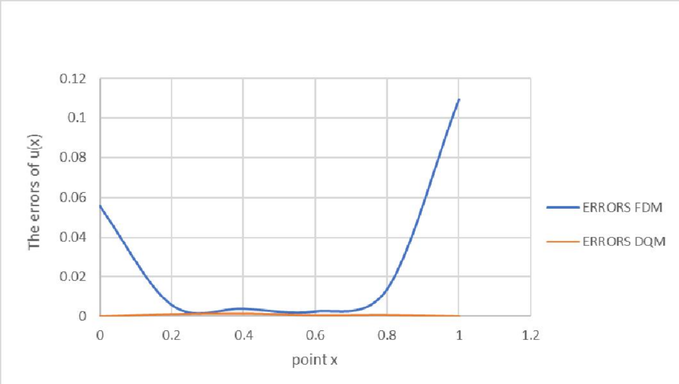

Figure 2: Graph of errors FDM and DQM against $t = 0.003$

The conclusion is from the tables and figures above from Case I and Case II, the arise from the Differential Quadrature Method showed reform than Finite Difference Method at each of time tested. In term of errors, the errors from the DQM results are smaller than the inference from the FDM. Therefore, the DQM is emending than FDM in stipulation of the correct solutions.

5. CONCLUSION

As a conclusion, all objective of this study has been fulfilled. To find the solution for the Burgers' Equation, there are various method can be applied and he Finite Different Method and Differential Quadrate Method are two of the method. For the researcher that wants to further the study on this field can use another method such as Runge-Kutta method, Method of Lines, Finite Element Method to find the solution of the Burgers' Equation. Lastly, the new researcher also can test the solution of the Burgers' Equation in term of stability.

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