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A STUDY OF FS-FUNCTIONS AND STUDY OF IMAGES OF FS-SUBSETS IN THE LIGHT OF REFINED DEFINITION OF IMAGES UNDER VARIOUS FS-FUNCTIONS

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Abstract: Vaddiparthi Yogewsara, G.Srinivas and Biswajit Rath introduced the concept of Fs-set ,Fs-subset, complement an of Fs-subset and proved important results like De Morgan laws for Fs-sets which are called Fs- De Morgan laws. In another paper[5] Vaddiparthi Yogeswara, Biswajit Rath and S.V.G.Reddy introduced the concept of Fs-Function between two Fs-subsets of a given Fs-set and defined an image of an Fs-subset under a given Fs-function. Also they studied the properties of images under various kinds of Fs-functions. In this paper we modify the definition of image of an Fs-subset under any given Fs-function and study the properties of images of Fs-subsets under various Fs-functions.

Keywords: Fs-set, Fs-subset, Fs-empty set, Fs-union, Fs-intersection, Fs-complement, Fs-De Morgan laws and Fs-Function and images of Fs-subsets

INTRODUCTION

Murthy[1] introduced F-set in order to prove Axiom of choice for fuzzy sets which is not true for L-fuzzy sets introduced by Goguen[2]. In the paper[3], Tridiv discussed fuzzy complement of an extended fuzzy subset and proved De Morgan laws etc. The extended Fuzzy set Tridiv considered contains the membership value $\mu_1(x)-\mu_2(x)$. $-\mu_2(x)$, a term in this expression will not be in the interval [0,1].To answer this incomprehensiveness, In the paper[4], Vaddiparthi Yogeswara , G.Srinivas and Biswajit Rath introduced the concept of Fs-set and developed the theory of Fs-sets in order to prove collection of all Fs-subsets of given Fs-set is a complete Boolean algebra under Fs-unions, Fs-intersections and Fs-complements. The Fs-sets they introduced contain Boolean valued membership functions .All most they are successful in their efforts in proving that result with some conditions. In another paper[5] Vaddiparthi Yogeswara, Biswajit Rath and S.V.G.Reddy introduced the concept of Fs-Function between two Fs-subsets of given Fs-set and defined an image of an Fs-subset under a given Fs-function. Also they studied the properties of images under various kinds of Fs-functions. In this paper we modify the definition of image of an Fs-subset under any given Fs-function and study the properties of images of Fs-subsets under various Fs-functions. For convenience of readers before beginning the paper, we mention various definitions and results in paper[4]. We denote the largest element of a complete Boolean algebra L_A[1.1] by M_A. We denote Fs-union and crisp set union by same symbol $\cup$ and similary Fs-intersection and crisp set intersection by the same symbol $\cap$.[X] denote the complete ideal generated by X and (X) denote the complete subalgebra generated by X in a complete Boolean algebra. For all lattice theoretic properties and Boolean algebraic properties we refer Szasz [7], Garret Birkhoff[8],Steven Givant • Paul Halmos[8] and Thomas Jech[9]

THEORY OF FS-SETS

1.1 Fs-set: Let U be a universal set, $A_1 \subseteq U$ and let $A \subseteq U$ be non-empty. A four tuple $\mathcal{A} = \left(A_1, A, \bar{A}\left(\mu_{1A_1}, \mu_{2A}\right), L_A\right)$ is said be an Fs-set if, and only if

(1) $A \subseteq A_1$

(2) L_A is a complete Boolean Algebra

(3) $\mu_{1A_1}: A_1 \longrightarrow L_A$, $\mu_{2A}: A \longrightarrow L_A$,are functions such that $\mu_{1A_1}|A \geq \mu_{2A}$

(4) $\bar{A}$:A⟶ L_A is defined by

$\bar{A}x = \mu_{1A_1}x \wedge (\mu_{2A}x)^c$,for each $x \in A$

1.2 Fs-subset

Let $\mathcal{A}=\left(A_1, A, \bar{A}\left(\mu_{1A_1}, \mu_{2A}\right), L_A\right)$ and $\mathcal{B}=\left(B_1, B, \bar{B}\left(\mu_{1B_1}, \mu_{2B}\right), L_B\right)$ be a pair of Fs-sets. $\mathcal{B}$ is said to be an Fs-subset of $\mathcal{A}$, denoted by $\mathcal{B} \subseteq \mathcal{A}$, if, and only if

(1) $B_1 \subseteq A_1$, $A \subseteq B$

(2) L_B is a complete subalgebra of L_A or $L_B \leq L_A$

(3) $\mu_{1B_1} \leq \mu_{1A_1}|B_1$, and $\mu_{2B}|A \geq \mu_{2A}$

1.3 Proposition: Let $\mathcal{B}$ and $\mathcal{A}$ be a pair of Fs-sets such that $\mathcal{B} \subseteq \mathcal{A}$. Then $\bar{B}x \leq \bar{A}x$ is true for each $x \in A$

1.4 Definition: For some L_X, such that $L_X \leq L_A$ a four tuple $\mathcal{X} = \left(X_1, X, \bar{X}\left(\mu_{1X_1}, \mu_{2X}\right), L_X\right)$ is not an Fs-set if, and only if

(a) $X \nsubseteq X_1$ or

(b) $\mu_{1X_1}x \ngeq \mu_{2X}x$, for some $x \in X \cap X_1$

Here onwards, any object of this type is called an Fs-empty set of first kind and we accept that it is an Fs-subset of $\mathcal{B}$ for any $\mathcal{B} \subseteq \mathcal{A}$.

Definition: An Fs-subset $\mathcal{Y}=\left(Y_1, Y, \bar{Y}\left(\mu_{1Y_1}, \mu_{2Y}\right), L_Y\right)$ of $\mathcal{A}$, is said to be an Fs-empty set of second kind if, and only if

(a') $Y_1 = Y = A$

(b') $L_Y \leq L_A$

(c') $\bar{Y} = 0$

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1.4.1 Remark: we denote Fs-empty set of first kind or Fs-empty set of second kind by $\Phi_{\mathcal{A}}$ and we prove later (1.15), $\Phi_{\mathcal{A}}$ is the least Fs-subset among all Fs-subsets of $\mathcal{A}$.

1.5 Definition: Let $\mathcal{B}_1 = (B_{11}, B_1, \bar{B}_1(\mu_{1B_{11}}, \mu_{2B_1}), L_{B_1})$ and $\mathcal{B}_2 = (B_{12}, B_2, \bar{B}_2(\mu_{1B_{12}}, \mu_{2B_2}), L_{B_2})$ be a pair of Fs-sets. We say that $\mathcal{B}_1$ and $\mathcal{B}_2$ are equal, denoted by $\mathcal{B}_1 = \mathcal{B}_2$ if, only if

[1] $B_{11} = B_{12}$, $B_1 = B_2$

[2] $L_{B_1} = L_{B_2}$

[3] (a) $(\mu_{1B_{11}} = \mu_{1B_{12}}$ and $\mu_{2B_1} = \mu_{2B_2})$,or (b) $\bar{B}_1 = \bar{B}_2$

1.5.1Remark: We can easily observed that 3(a) and 3(b) not equivalent statements.

1.6 Proposition: $\mathcal{B}_1 = (B_{11}, B_1, \bar{B}_1(\mu_{1B_{11}}, \mu_{B_1}), L_{B_1})$ *and* $\mathcal{B}_2 = (B_{12}, B_2, \bar{B}_2(\mu_{1B_{12}}, \mu_{B_2}), L_{B_2})$ *are equal if, only if* $\mathcal{B}_1 \subseteq \mathcal{B}_2$ and $\mathcal{B}_2 \subseteq \mathcal{B}_1$

1.7 Definition of Fs-union for a given pair of Fs-subsets of $\mathcal{A}$:

Let $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ and $\mathcal{C}= (C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$, be a pair of Fs-subsets of $\mathcal{A}$. Then,

the Fs-union of $\mathcal{B}$ and $\mathcal{C}$,denoted by $\mathcal{B}\cup\mathcal{C}$ is defined as $\mathcal{B}\cup\mathcal{C}=\mathcal{D}=(D_1, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D)$,where

(1) $D_1 = B_1 \cup C_1$, $D = B \cap C$

(2) $L_D = L_B \vee L_C$=complete subalgebra generated by $L_B \cup L_C$

(3) $\mu_{1D_1}: D_1 \longrightarrow L_D$ is defined by

$\mu_{1D_1}x = (\mu_{1B_1} \vee \mu_{1C_1})x$

$\mu_{2D} : D \longrightarrow L_D$ is defined by

$\mu_{2D}x = \mu_{2B}x \wedge \mu_{2C}x$

$\bar{D}: D \longrightarrow L_D$ is defined by

$\bar{D}x = \mu_{1D_1}x\wedge(\mu_{2D}x)^c$

1.8 Proposition: *$\mathcal{B}\cup\mathcal{C}$ is an Fs-subset of $\mathcal{A}$.*

1.9 Definition of Fs-intersection for a given pair of Fs-subsets of $\mathcal{A}$:

Let $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ and $\mathcal{C}= (C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ be a pair of Fs-subsets of $\mathcal{A}$ satisfying the following conditions:

(i) $B_1 \cap C_1 \supseteq B \cup C$

(ii) $\mu_{1B_1}x \wedge \mu_{1C_1}x \geq (\mu_{2B} \vee \mu_{2C})x$,for each $x \in A$

Then, the Fs-intersection of $\mathcal{B}$ and $\mathcal{C}$, denoted by $\mathcal{B}\cap\mathcal{C}$ is defined as

$\mathcal{B}\cap\mathcal{C} =\mathcal{E}=(E_1, E, \bar{E}(\mu_{1E_1}, \mu_{2E}), L_E)$,where

(a) $E_1 = B_1 \cap C_1$, $E = B \cup C$

(b) $L_E = L_B \wedge L_C$=$L_B \cap L_C$

(c) $\mu_{1E_1}: E_1 \longrightarrow L_E$ is defined by $\mu_{1E_1}x = \mu_{1B_1}x \wedge \mu_{1C_1}x$

$\mu_{2E} : E \longrightarrow L_E$ is defined by

$\mu_{2E}x = (\mu_{2B} \vee \mu_{2C})x$

$\bar{E}: E \longrightarrow L_E$ is defined by

$\bar{E}x = \mu_{1E_1}x\wedge(\mu_{2E}x)^c$.

1.9.1 Remark: If (i) or (ii) fails we define $\mathcal{B}\cap\mathcal{C}$ as $\mathcal{B}\cap\mathcal{C}=\Phi_{\mathcal{A}}$, which is the Fs-empty set of first kind.

1.10 Proposition: *For any pair of Fs-subsets* $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ *and* $\mathcal{C}=(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ *of $\mathcal{A}$, the following results are true*

(1) $\mathcal{B}\subseteq\mathcal{B}\cup\mathcal{C}$ and $\mathcal{C}\subseteq\mathcal{B}\cup\mathcal{C}$

(2) $\mathcal{B}\cap\mathcal{C}\subseteq\mathcal{B}$ and $\mathcal{B}\cap\mathcal{C}\subseteq\mathcal{C}$ provided $\mathcal{B}\cap\mathcal{C}$ exists

(3) $\mathcal{B}\subseteq\mathcal{C}$ implies $\mathcal{B}\cup\mathcal{C}=\mathcal{C}$

(4) $\mathcal{B}\cap\mathcal{C}=\mathcal{B}$ when $\mathcal{B}\neq \Phi_{\mathcal{A}}$ and $\mathcal{B}\subseteq\mathcal{C}$ and $\Phi_{\mathcal{A}} \cap\mathcal{C}=\Phi_{\mathcal{A}}$

(5) $\mathcal{B}\cup\mathcal{C}= \mathcal{C} \cup \mathcal{B}$ (commutative law of Fs-union)

(6) $\mathcal{B}\cap\mathcal{C}= \mathcal{C} \cap \mathcal{B}$ provided $\mathcal{B}\cap\mathcal{C}$ exists. (commutative law of Fs-intersection)

(7) $\mathcal{B}\cup\mathcal{B}=\mathcal{B}$

(8) $\mathcal{B}\cap\mathcal{B}=\mathcal{B}$ ((7)and (8) are Idempotent laws of Fs-union and Fs-intersection respectively)

1.11 Proposition: For any Fs-subsets $\mathcal{B}$, $\mathcal{C}$and $\mathcal{D}$ of $\mathcal{A} = (A_1, A, \bar{A}(\mu_{1A_1}, \mu_{2A}), L_A)$,

the following associative laws are true:

(I) $\mathcal{B} \cup (\mathcal{C} \cup \mathcal{D}) = (\mathcal{B} \cup \mathcal{C}) \cup \mathcal{D}$

(II) $\mathcal{B} \cap (\mathcal{C} \cap \mathcal{D}) = (\mathcal{B} \cap \mathcal{C}) \cap \mathcal{D}$, whenever Fs-intersections exist.

1.12 Arbitrary Fs-unions and arbitrary Fs-intersections:

Given a family $(\mathcal{B}_i)_{i\in I}$ of Fs-subsets of $\mathcal{A}=(A_1, A, \bar{A}(\mu_{1A_1}, \mu_{2A}), L_A)$, where $\mathcal{B}_i = (B_{1i}, B_i, \bar{B}_i(\mu_{1B_{1i}}, \mu_{2B_i}), L_{B_i})$,for any $i \in I$

1.13 Definition of Fs-union is as follows

Case (1): For I=Φ,define Fs-union of $(\mathcal{B}_i)_{i\in I}$,denoted by $\bigcup_{i\in I}\mathcal{B}_i$ as $\bigcup_{i\in I}\mathcal{B}_i = \Phi_{\mathcal{A}}$, which is the Fs-empty set

Case (2): Define for I≠Φ, Fs-union of $(\mathcal{B}_i)_{i\in I}$ denoted by $\bigcup_{i\in I}\mathcal{B}_i$ as follow

$$\bigcup_{i\in I}\mathcal{B}_i = \mathcal{B} = (B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B),$$

where

(a) $B_1 = \bigcup_{i\in I} B_{1i}$,$B = \bigcap_{i\in I} B_i$

(b) $L_B = \bigvee_{i\in I} L_{B_i}$ = complete subalgebra generated by $\bigcup L_i (L_i = L_{B_i})$

(c) $\mu_{1B_1}: B_1 \rightarrow L_B$ is defined by

$\mu_{1B_1}x = (\bigvee_{i\in I}\mu_{1B_{1i}})x = \bigvee_{i\in I_x}\mu_{1B_{1i}}x$, where $I_x = \{i \in I | x \in B_i\}$

$\mu_{2B}: B \longrightarrow L_B$ is defined by $\mu_{2B}x = (\bigwedge_{i\in I}\mu_{2B_i})x = \bigwedge_{i\in I}\mu_{2B_i}x$

$\bar{B}: B \longrightarrow L_B$ is defined by $\bar{B}x = \mu_{1B_1}x\wedge(\mu_{2B}\, x)^c$

1.13.1Remark: We can easily show that (d) $B_1 \supseteq B$ and $\mu_{1B_1}|B \geq \mu_{2B}$.

1.14 Definition of Fs-intersection:

Case (1): For I=Φ, we define Fs-intersection of $(\mathcal{B}_i)_{i\in I}$, denoted by $\bigcap_{i\in I}\mathcal{B}_i$ as $\bigcap_{i\in I}\mathcal{B}_i = \mathcal{A}$

Case (2): Suppose $\bigcap_{i\in I} B_{1i} \supseteq \bigcup_{i\in I} B_i$ and $\bigwedge_{i\in I}\mu_{1B_{1i}}|(\bigcup_{i\in I} B_i) \geq \bigvee_{i\in I}\mu_{2B_i}$

Then, we define Fs-intersection of $(\mathcal{B}_i)_{i\in I}$, denoted by $\bigcap_{i\in I}\mathcal{B}_i$ as follows

$$\bigcap_{i\in I}\mathcal{B}_i = \mathcal{C} = (C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$$

(a') $C_1 = \bigcap_{i\in I} B_{1i}$, $C = \bigcup_{i\in I} B_i$

(b') $L_C = \bigwedge_{i\in I} L_{B_i}$

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(c') $\mu_{1C_1}: C_1 \longrightarrow L_C$ is defined by
$\mu_{1C_1}x = (\wedge_{i\in I}\mu_{1B_{1i}})x = \wedge_{i\in I}\mu_{1B_{1i}}x$
$\mu_{2C}: C \longrightarrow L_C$ is defined by
$\mu_{2C}x = (\vee_{i\in I}\mu_{2B_i})x = \vee_{i\in I_x}\mu_{2B_i}x$,
where, $I_x = \{i \in I \mid x \in B_i\}$
$\bar{C}: C \longrightarrow L_C$ is defined by $\bar{C}x = \mu_{1C_1}x \wedge (\mu_{2C}\,x)^c$

Case (3): $\cap_{i\in I} B_{1i} \not\supseteq \cup_{i\in I} B_i$ or $\wedge_{i\in I}\mu_{1B_{1i}}|(\cup_{i\in I}B_i) \not\geq \vee_{i\in I}\mu_{2B_i}$
We define

$$\bigcap_{i\in I}\mathcal{B}_i = \Phi_{\mathcal{A}}$$

1.14.1Lemma: For any Fs-subset $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_A)$ and $\mathcal{B} \subseteq \mathcal{B}_i = (B_{1i}, B_i, \bar{B}_i(\mu_{1B_{1i}}, \mu_{2B_i}), L_{B_i})$

1.15 Proposition: $(\mathcal{L}(\mathcal{A}), \cap)$ is $\wedge$-complete lattics.

1.15.1 Corollary: For any Fs-subset $\mathcal{B}$ of $\mathcal{A}$, the following results are true
(i) $\Phi_{\mathcal{A}} \cup \mathcal{B} = \mathcal{B}$
(ii) $\Phi_{\mathcal{A}} \cap \mathcal{B} = \Phi_{\mathcal{A}}$.

1.16 Proposition: $(\mathcal{L}(\mathcal{A}), \cup)$ is $\vee$-complete lattics.

1.16.1 Corollary: $(\mathcal{L}(\mathcal{A}), \cup, \cap)$ is a complete lattice with $\vee$ and $\wedge$

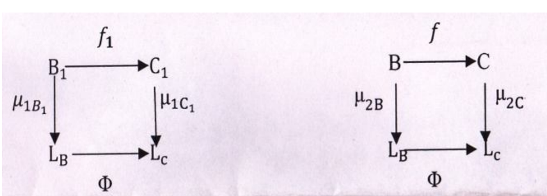
1.17 Proposition: Let $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$, $\mathcal{C}=(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ and $\mathcal{D}=(D_1, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D)$. Then $\mathcal{B} \cup (\mathcal{C} \cap \mathcal{D}) = (\mathcal{B} \cup \mathcal{C}) \cap (\mathcal{B} \cup \mathcal{D})$ provided $\mathcal{C} \cap \mathcal{D}$ exists.

1.18 Proposition: Let $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$, $\mathcal{C}=(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ and $\mathcal{D}=(D_1, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D)$. Then $\mathcal{B} \cap (\mathcal{C} \cup \mathcal{D}) = (\mathcal{B} \cap \mathcal{C}) \cup (\mathcal{B} \cap \mathcal{D})$ provided in R.H.S $(\mathcal{B} \cap \mathcal{C})$ and $(\mathcal{B} \cap \mathcal{D})$ exist.

THEORY OF FS-FUNCTIONS

2.1 Fs-Function

A Triplet (f_1, f, Φ) is said to be is an Fs-Function between two given Fs-subsets $\mathcal{B} = (B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ and $\mathcal{C} = (C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ of $\mathcal{A}$, denoted by $(f_1, f, \Phi): \mathcal{B} = (B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B) \longrightarrow \mathcal{C} = (C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ if, and only if (using the diagrams) .



(Fig-1:Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$)

(1a) $f_1|_B = f$ is onto
(1b) $\Phi: L_B \longrightarrow L_c$ is complete homomorphism
In general (f_1, f, Φ) is denoted by $\bar{f}$

2.2 Proposition: (i) $\mu_{1C_1}|_C \circ f_1|_B \geq \mu_{2C} \circ f$
(ii) $\Phi \circ \mu_{1B_1}|_B \geq \Phi \circ \mu_{2B}$

Proof (i): $f_1 x = f$, for each $x \in B$
$(\mu_{1C_1}|_C \circ f_1|_B)x = \mu_{1C_1}(f_1 x) = \mu_{1C_1}(fx) \geq \mu_{2C}(fx) = (\mu_{2C} \circ f)x$
Hence $\mu_{1C_1}|_C \circ f_1|_B \geq \mu_{2C} \circ f$.

Proof (ii): $\mu_{1B_1}x \geq \mu_{2B}x$
$\Longrightarrow \Phi(\mu_{1B_1}x) \geq \Phi(\mu_{2B}x)$
($\because \Phi$ is a complete homomorphism)
$\Longrightarrow (\Phi \circ \mu_{1B_1})x \geq (\Phi \circ \mu_{2B})x$
Hence $\Phi \circ \mu_{1B_1}|_B \geq \Phi \circ \mu_{2B}$

2.2.1 Remark: Φ is a complete homomorphism between complete Boolean algebras implies $\Phi(0) = 0$ and $\Phi(1) = 1$ and $[\Phi(a)]^c = \Phi(a^c)$
Therefore $\Phi(a) \wedge \Phi(a^c) = \Phi(a \wedge a^c) = \Phi(0) = 0$
$\Phi(a) \vee \Phi(a^c) = \Phi(a \vee a^c) = \Phi(1)=1$

2.3 Def: Increasing Fs-function

$\bar{f}$ is said to be an increasing Fs- function, and denoted by $\bar{f}_i$ if ,and only if(using fig-1)
(2a) $\mu_{1C_1}|_C \circ f_1|_B \geq \Phi \circ \mu_{1B_1}$
(2b) $\mu_{2C} \circ f \leq \Phi \circ \mu_{2B}$

2.4 Proposition: $\Phi \circ (\mu_{2B}x)^c = [(\Phi \circ \mu_{2B})x]^c$
Proof: LHS: $\Phi \circ (\mu_{2B}x)^c = \Phi[(\mu_{2B}x)^c] = [\Phi(\mu_{2B}x)]^c = [(\Phi \circ \mu_{2B})x]^c$

2.5 Proposition: $\Phi \circ \bar{B} \leq \bar{C} \circ f$,provided $\bar{f}$ is an increasing Fs-function
Proof: $\Phi(\bar{B}x) = \Phi(\mu_{1B_1}x \wedge (\mu_{2B}x)^c)$
$= \Phi(\mu_{1B_1}x) \wedge \Phi[(\mu_{2B}x)^c]$
$= \Phi(\mu_{1B_1}x) \wedge [\Phi(\mu_{2B}x)]^c$
$= (\Phi \circ \mu_{1B_1})x \wedge [(\Phi \circ \mu_{2B})x]^c \leq (\mu_{1C_1} \circ f_1)x \wedge [(\mu_{2C} \circ f)x]^c = \mu_{1C_1}(f_1 x) \wedge [\mu_{2C}(fx)]^C$
$= \mu_{1C_1}(fx) \wedge [\mu_{2C}(fx)]^C = \bar{C}(fx)$
Hence $\Phi \circ \bar{B} \leq \bar{C} \circ f$

2.6 Def: Decreasing Fs-function

$\bar{f}$ is said to be decreasing Fs-function denoted as $\bar{f}_d$ and if and only if
(3a) $\mu_{1C_1}|_C \circ f_1|_B \leq \Phi \circ \mu_{1B_1}$
(3b) $\mu_{2C} \circ f \geq \Phi \circ \mu_{2B}$

2.7 Proposition: $\Phi \circ \bar{B} \geq \bar{C} \circ f$,provided $\bar{f}$ is a decreasing Fs-function
Proof: $\Phi(\bar{B}x) = \Phi(\mu_{1B_1}x \wedge (\mu_{2B}x)^c)$
$= \Phi(\mu_{1B_1}x) \wedge \Phi[(\mu_{2B}x)^c]$
$= \Phi(\mu_{1B_1}x) \wedge [\Phi(\mu_{2B}x)]^c$
$= (\Phi \circ \mu_{1B_1})x \wedge [(\Phi \circ \mu_{2B})x]^c \geq (\mu_{1C_1} \circ f_1)x \wedge [(\mu_{2C} \circ f)x]^c = \mu_{1C_1}(f_1 x) \wedge [\mu_{2C}(fx)]^C$
$= \mu_{1C_1}(fx) \wedge [\mu_{2C}(fx)]^C = \bar{C}(fx)$
Hence $\Phi \circ \bar{B} \geq \bar{C} \circ f$

2.8 Def:Preserving Fs- function

$\bar{f}$ is said to be preserving Fs-function and denoted as $\bar{f}_p$ if ,and only if
(4a) $\mu_{1C_1}|_C \circ f_1|_B = \Phi \circ \mu_{1B_1}$
(4b) $\mu_{2C} \circ f = \Phi \circ \mu_{2B}$

2.9 Proposition: $\Phi \circ \bar{B} = \bar{C} \circ f$,provided $\bar{f}$ is Fs- preserving function

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Proof: $\Phi(\bar{B}x) = \Phi\left(\mu_{1B_1}x \wedge (\mu_{2B}x)^c\right)$
$= \Phi\left(\mu_{1B_1}x\right) \wedge \Phi[(\mu_{2B}x)^c]$
$= \Phi\left(\mu_{1B_1}x\right) \wedge [\Phi(\mu_{2B}x)]^c$
$= \left(\Phi \circ \mu_{1B_1}\right)x \wedge [(\Phi \circ \mu_{2B})x]^c$
$= \left(\mu_{1C_1} \circ f_1\right)x \wedge [(\mu_{2C} \circ f)x]^c$
$= \mu_{1C_1}(f_1x) \wedge [\mu_{2C}(fx)]^C$
$= \mu_{1C_1}(fx) \wedge [\mu_{2C}(fx)]^C = \bar{C}(fx)$
Hence $\Phi \circ \bar{B} = \bar{C} \circ f$

2.10 Def: Composition of two Fs-function

Given two Fs-functions $\bar{f}: \mathcal{B} \to \mathcal{C}$ and $\bar{g}: \mathcal{C} \to \mathcal{D}$.We denote composition of $\bar{g}$ and $\bar{f}$ as $\bar{g} \circ \bar{f}$ and define as $\left(\bar{g} \circ \bar{f}\right) = (g_1, g, \Psi) \circ (f_1, f, \Phi) = [g_1 \circ f_1, g \circ f, \Psi \circ \Phi]$

2.11 Proposition: Composition of two increasing Fs-function are increasing.

Proof: suppose $\bar{f}_i: \left(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B\right) \longrightarrow \left(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C\right)$ and $\bar{g}_i: \left(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C\right) \longrightarrow \left(D_1, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D\right)$ are two increasing Fs-functions

Implies (1) $\mu_{1C_1}|_C \circ f_1|_B \geq \Phi \circ \mu_{1B_1}$
(2) $\mu_{2C} \circ f \leq \Phi \circ \mu_{2B}$
And (3) $\mu_{1D_1}|_D \circ g_1|_c \geq \Psi \circ \mu_{1c_1}$
(4) $\mu_{2D} \circ g \leq \Psi \circ \mu_{2C}$
Need to prove that
(5) $\mu_{1D_1}|_D \circ g_1|_c f_1|_B \geq (\Psi \circ \Phi) \circ \mu_{1B_1}$
(6) $\mu_{2D} \circ gf \leq (\Psi \circ \Phi) \circ \mu_{2C}$

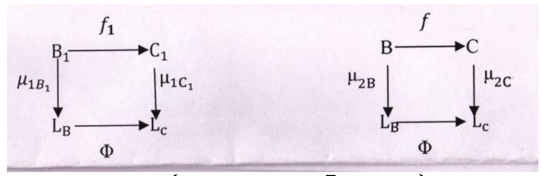
Proof (5): $\left(\mu_{1D_1}|_D \circ g_1|_c f_1|_B\right)x$
$= \left(\mu_{1D_1}|_D \circ (g_1|_c \circ f_1|_B)\right)x$
$= \left((\mu_{1D_1}|_D \circ g_1|_c) \circ f_1|_B\right)x$
$= \left(\mu_{1D_1}|_D \circ g_1|_c\right)(f_1x)$
$\geq \left(\Psi \circ \mu_{1c_1}\right)(f_1x)$
$= \Psi\left(\mu_{1c_1}(f_1x)\right) = \Psi\left[\left(\mu_{1C_1} \circ f_1|_B\right)\right]x$ ($\because \Psi$ is a homomorphism)
$\geq \Psi\left[\left(\Phi \circ \mu_{1B_1}\right)\right]x = \left[\Psi \circ \left(\Phi \circ \mu_{1B_1}\right)\right]x$
$= \left[(\Psi \circ \Phi) \circ \mu_{1B_1}\right]x$
Hence $\mu_{1D_1}|_D \circ g_1|_c f_1|_B \geq (\Psi \circ \Phi) \circ \mu_{1B_1}$
Proof (6): $[\mu_{2D} \circ (g \circ f)]x$
$= [(\mu_{2D} \circ g) \circ f]x = (\mu_{2D} \circ g)(fx)$
$\leq (\Psi \circ \mu_{2C})(fx)$
$= \Psi\left(\mu_{2C}(fx)\right) = \Psi[(\mu_{2C} \circ f)x]$
$\leq \Psi[(\Phi \circ \mu_{2B})x] = [\Psi \circ (\Phi \circ \mu_{2B})]x = [(\Psi \circ \Phi) \circ \mu_{2B}]x$
Hence $\mu_{2D} \circ gf \leq (\Psi \circ \Phi) \circ \mu_{2C}$
Hence $(g_1, g, \Psi)_i \circ (f_1, f, \Phi)_i = [g_1 \circ f_1, g \circ f, \Psi \circ \Phi]_i$

2.12 Proposition: Composition of two decreasing Fs-function are decreasing.

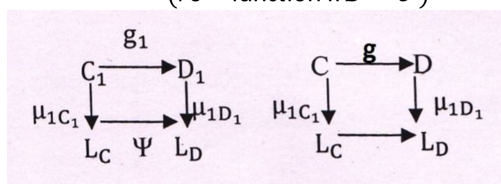
Proof: suppose $\bar{f}_d: \left(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B\right) \longrightarrow \left(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C\right)$ and $\bar{g}_d: \left(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C\right) \longrightarrow \left(D_1, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D\right)$ are two decreasing functions

Implies (a) $\mu_{1C_1}|_C \circ f_1|_B \leq \Phi \circ \mu_{1B_1}$
(b) $\mu_{2C} \circ f \geq \Phi \circ \mu_{2B}$
And (c) $\mu_{1D_1}|_D \circ g_1|_c \leq \Psi \circ \mu_{1c_1}$
(d) $\mu_{2D} \circ g \geq \Psi \circ \mu_{2C}$
Need to prove that

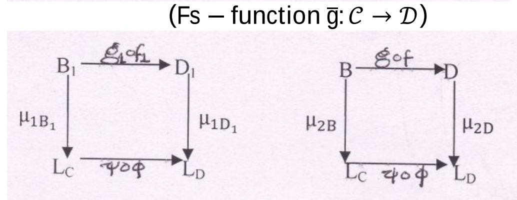
(e)$\mu_{1D_1}|_D \circ (g_1|_c \circ f_1|_B) \leq (\Psi \circ \Phi) \circ \mu_{1B_1}$
(f) $\mu_{2D} \circ (g \circ f) \geq (\Psi \circ \Phi) \circ \mu_{2C}$



$\left(\text{Fs} - \text{function } \bar{f}: \mathcal{B} \to \mathcal{C}\right)$



$\left(\text{Fs} - \text{function } \bar{g}: \mathcal{C} \to \mathcal{D}\right)$



$\left(\text{Fig 2: Compsition of } \bar{f} \text{ and } \bar{g} \text{ i.e. } (\bar{g} \circ \bar{f}): \mathcal{B} \to \mathcal{D}\right)$

Proof (e): $\left(\mu_{1D_1}|_D \circ g_1|_c f_1|_B\right)x$
$= \left(\mu_{1D_1}|_D \circ (g_1|_c \circ f_1|_B)\right)x$
$= \left((\mu_{1D_1}|_D \circ g_1|_c) \circ f_1|_B\right)x$
$= \left(\mu_{1D_1}|_D \circ g_1|_c\right)(f_1x) \leq \left(\Psi \circ \mu_{1c_1}\right)(f_1x)$
$= \Psi\left(\mu_{1c_1}(f_1x)\right) = \Psi\left[\left(\mu_{1C_1} \circ f_1|_B\right)\right]x$ ($\because \Psi$ is a homomorphism)
$\leq \Psi\left[\left(\Phi \circ \mu_{1B_1}\right)\right]x = \left[\Psi \circ \left(\Phi \circ \mu_{1B_1}\right)\right]x$
$= \left[(\Psi \circ \Phi) \circ \mu_{1B_1}\right]x$
Hence $\mu_{1D_1}|_D \circ g_1|_c f_1|_B \leq (\Psi \circ \Phi) \circ \mu_{1B_1}$
Proof (f): $[\mu_{2D} \circ (g \circ f)]x$
$= [(\mu_{2D} \circ g) \circ f]x$
$= (\mu_{2D} \circ g)(fx) \geq (\Psi \circ \mu_{2C})(fx)$
$= \Psi\left(\mu_{2C}(fx)\right) = \Psi[(\mu_{2C} \circ f)x]$
$\geq \Psi[(\Phi \circ \mu_{2B})x] = [\Psi \circ (\Phi \circ \mu_{2B})]x$
$= [(\Psi \circ \Phi) \circ \mu_{2B}]x$
Hence $\mu_{2D} \circ (g \circ f) \geq (\Psi \circ \Phi) \circ \mu_{2C}$
Hence $(g_1, g, \Psi)_d \circ (f_1, f, \Phi)_d = [g_1 \circ f_1, g \circ f, \Psi \circ \Phi]_d$

2.13 Proposition: Composition of two preserving Fs-function are preserving.

Proof: suppose $\bar{f}_p: \left(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B\right) \longrightarrow \left(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C\right)$ and $\bar{g}_p: \left(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C\right) \longrightarrow \left(D_1, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D\right)$are two preserving functions

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Implies (a) $\mu_{1C_1}|_C \circ f_1|_B = \Phi \circ \mu_{1B_1}$
(b) $\mu_{2C} \circ f = \Phi \circ \mu_{2B}$

And (c) $\mu_{1D_1}|_D \circ g_1|_c = \Psi \circ \mu_{1c_1}$
(d) $\mu_{2D} \circ g = \Psi \circ \mu_{2C}$

Need to prove that

(e) $\mu_{1D_1}|_D \circ (g_1|_c \circ f_1|_B) = (\Psi \circ \Phi) \circ \mu_{1B_1}$
(f) $\mu_{2D} \circ (g \circ f) = (\Psi \circ \Phi) \circ \mu_{2C}$
Proof (e): $(\mu_{1D_1}|_D \circ (g_1|_c \circ f_1|_B))x$
$= ((\mu_{1D_1}|_D \circ g_1|_C) \circ f_1|_B)x$
$= (\mu_{1D_1}|_D \circ g_1|_C)(f_1x) = (\Psi \circ \mu_{1c_1})(f_1x)$
$= \Psi\left(\mu_{1c_1}(f_1x)\right) = \Psi[(\mu_{1C_1} \circ f_1|_B)]x$ ($\because \Psi$ is a homomorphism)
$= \Psi[(\Phi \circ \mu_{1B_1})]x = [\Psi \circ (\Phi \circ \mu_{1B_1})]x$
$= [(\Psi \circ \Phi) \circ \mu_{1B_1}]x$
Hence
$\mu_{1D_1}|_D \circ (g_1|_c \circ f_1|_B) = (\Psi \circ \Phi) \circ \mu_{1B_1}$
Proof (f): $[\mu_{2D} \circ (g \circ f)]x$
$= [(\mu_{2D} \circ g) \circ f]x$
$= (\mu_{2D} \circ g)(fx) = (\Psi \circ \mu_{2C})(fx)$
$= \Psi(\mu_{2C}(fx)) = \Psi[(\mu_{2C} \circ f)x]$
$= \Psi[(\Phi \circ \mu_{2B})x] = [\Psi \circ (\Phi \circ \mu_{2B})]x = [(\Psi \circ \Phi) \circ \mu_{2B}]x$
Hence $\mu_{2D} \circ (g \circ f) = (\Psi \circ \Phi) \circ \mu_{2C}$
Hence $(g_1, g, \Psi)_{p[} \circ (f_1, f, \Phi)_p = [g_1 \circ f_1, g \circ f, \Psi \circ \Phi]_p$

2.13.1 Remark: (f_1, f, Φ) is preserving if, and only if (f_1, f, Φ) simultaneously both increasing and decreasing

2.14 Proposition: The class of all Fs-sets as objects together with morphism sets Fs-functions under the partial operation denoted by ∘ is called composition between Fs-functions whenever it exists is a category denoted by 𝔽s-SET

Where $(g_1, g, \Psi) \circ (f_1, f, \Phi) = (g_1 \circ f_1, g \circ f, \Psi \circ \Phi)$

Proof: Given objects $(B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ and $(C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ with an Fs-function $(f_1, f, \Phi): \mathcal{B} = (B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B) \longrightarrow \mathcal{C} = (C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$

We can easily show that

(5a) $(f_1, f, \Phi) \circ (1_{B_1}, 1_B, 1_{L_B}) = (f_1, f, \Phi)$
(5b) $(1_{C_1}, 1_C, 1_{L_C}) \circ (f_1, f, \Phi) = (f_1, f, \Phi)$

Where $(1_{B_1}, 1_B, 1_{L_B}): (B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B) \longrightarrow (B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ is identity Fs-function, where $1_{B_1}: B_1 \longrightarrow B_1$, $1_B: B \longrightarrow B$ and $1_{L_B}: L_B \longrightarrow L_B$ are identity functions

(2) For any given Fs-sets $(B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$, $(C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$, $(D_1, D, \overline{D}(\mu_{1D_1}, \mu_{2D}), L_D)$ and $(E_1, E, \overline{E}(\mu_{1E_1}, \mu_{2E}), L_E)$ and Fs-functions
$(f_1, f, \Phi_1): (B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B) \longrightarrow (C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$
$(g_1, g, \Phi_2): (C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C) \longrightarrow (D_1, D, \overline{D}(\mu_{1D_1}, \mu_{2D}), L_D)$
$(h_1, h, \Phi_3): (D_1, D, \overline{D}(\mu_{1D_1}, \mu_{2D}), L_D) \longrightarrow (E_1, E, \overline{E}(\mu_{1E_1}, \mu_{2E}), L_E)$

We can easily show that

$[(h_1, h, \Phi_3) \circ (g_1, g, \Phi_2)] \circ (f_1, f, \Phi_1) = (h_1, h, \Phi_3) \circ [(g_1, g, \Phi_2) \circ (f_1, f, \Phi_1)]$

2.15 Proposition: The class of all Fs-sets as objects together with morphism sets increasing Fs-functions under the partial operation denoted by ∘ is called composition between increasing Fs-functions whenever it exists is a category denoted by 𝔽s-SET$_i$

2.16 Proposition: The class of all Fs-sets as objects together with morphism sets decreasing Fs-functions under the partial operation denoted by ∘ is called composition between decreasing Fs-functions whenever it exists is a category denoted by 𝔽s-SET$_d$

2.17 Proposition: The class of all Fs-sets as objects together with morphism sets preserving Fs-functions under the partial operation denoted by ∘ is called composition between preserving Fs-functions whenever it exists is a category denoted by 𝔽s-SET$_p$

IMAGES OF FS-SUBSETS UNDER FS-FUNCTION

2.18 Def: Fs-image of an Fs-subset Fs-function:
Let $\overline{f}: (B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B) \longrightarrow (C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$

Let
$\mathcal{D} = (D_1, D, \overline{D}(\mu_{1D_1}, \mu_{2D}), L_D) \subseteq \mathcal{B} = (B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ then

(a) $D_1 \subseteq B_1$, $B \subseteq D$
(b) $L_D \leq L_B$
(c) $(\mu_{1D_1} \leq \mu_{1B_1}|D_1$, and $\mu_{2D}|B \geq \mu_{2B})$ or $\overline{D}x \leq \overline{B}x$ for each $x \in B$

Define $\overline{f}(\mathcal{D}) = \mathcal{E} = (E_1, E, \overline{E}(\mu_{1E_1}, \mu_{2E})L_E)$, where

(d) $E_1 = f_1(D_1)$
(e) $E = f_1(D)$
(f) $L_E = ([X] \cup \Phi L_D)$, [X] is complete ideal generated by $X = \{\mu_{1C_1}y | y \in E_1, y = f_1x, x \in D_1\}$,
(g) $\mu_{1E_1}: E_1 \longrightarrow L_E$ is define by

$$\mu_{1E_1}y = \mu_{2C} \vee \left[\mu_{1C_1} \wedge \left(\bigvee_{\substack{y=f_1x \\ x \in D_1}} \Phi \mu_{1D_1} x\right)\right]$$

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(h) $\mu_{2E}: E \longrightarrow L_E$ is define by
$\mu_{2E}y = \mu_{2C}\vee\left[\mu_{1C_1}\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in D}}\Phi\mu_{2D}x\right)\right]$

2.19 Propositions:$\bar{f}(\mathcal{D})$ is an Fs-subset of $\mathcal{C}=\left(C_1, C, \bar{C}(\mu_{1C_1},\mu_{2C}), L_C\right)$

Proof: $\bar{f}(\mathcal{D}) = \mathcal{E} = \left(E_1, E, \bar{E}(\mu_{1E_1},\mu_{2E})L_E\right)$, where

[1] $E_1 = f_1(D_1) \subseteq C_1$
[2] $E = f_1(D) \supseteq f_1(B) = f(B) = C$ ($\because$ f is onto)
[3] $L_E = ([X] \cup \Phi L_D)$, $[X]$ is complete ideal generated by $X = \{\mu_{1C_1}y \mid y \in E_1, y = f_1x, x \in D_1\}$
$\Longrightarrow L_E \leq L_C$
[4] $\mu_{1E_1}: E_1 \longrightarrow L_E$ is define by
$\mu_{1E_1}y = \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in D_1}}\Phi\mu_{1D_1}x\right)\right] \leq \mu_{1C_1}y$
[5] $\mu_{2E}: E \longrightarrow L_E$ is define by
$\mu_{2E}y = \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in D}}\Phi\mu_{2D}x\right)\right] \geq \mu_{2C}y$

Hence all the above implies $\bar{f}(\mathcal{D})$ is an Fs-subset of $\mathcal{C}$

2.20 Proposition: $\bar{f}: \left(B_1, B, \bar{B}(\mu_{1B_1},\mu_{2B}), L_B\right) \longrightarrow \left(C_1, C, \bar{C}(\mu_{1C_1},\mu_{2C}), L_C\right)$and for any pair of Fs-subsets $\mathcal{H}_1 = \left(H_{11}, H_1, \bar{H}_1(\mu_{1H_{11}},\mu_{2H_1}), L_{H_1}\right)$ and $\mathcal{H}_2 = \left(H_{12}, H_2, \bar{H}_2(\mu_{1H_{12}},\mu_{2H_2}), L_{H_2}\right)$ of $\mathcal{B} = \left(B_1, B, \bar{B}(\mu_{1B_1},\mu_{2B}), L_B\right)$ such that $\mathcal{H}_1 \subseteq \mathcal{H}_2$,then

$$\bar{f}(\mathcal{H}_1) \subseteq \bar{f}(\mathcal{H}_2)$$

Proof: Suppose
$\bar{f}(\mathcal{H}_1) = \mathcal{G}_1 = \left(G_{11}, G_1, \bar{G}_1(\mu_{1G_{11}},\mu_{2G_1})L_{G_1}\right)$,where

(a) $G_{11} = f_1(H_{11})$
(b) $G_1 = f_1(H_1)$
(c) $L_{G_1} = \left([X_1] \cup \Phi L_{H_1}\right)$, $[X_1]$ is complete ideal generated by $X_1 = \{\mu_{1C_1}y \mid y \in G_{11}, y = f_1x, x \in H_{11}\}$
(d) $\mu_{1G_{11}}: G_{11} \longrightarrow L_{G_1}$ is defined by $\mu_{1G_{11}}y = \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_{11}}}\Phi\mu_{1H_{11}}x\right)\right]$
(e) $\mu_{2G_1}: G_1 \longrightarrow L_{G_2}$ is defined by $\mu_{2G_1}y = \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_1}}\Phi\mu_{2H_1}x\right)\right]$

Again suppose
$\bar{f}(\mathcal{H}_2) = \mathcal{G}_2 = \left(G_{12}, G_2, \bar{G}_2(\mu_{1G_{12}},\mu_{2G_2})L_{G_2}\right)$,where

(f) $G_{12} = f_1(H_{12})$
(g) $G_2 = f_1(H_2)$
(h) $L_{G_2} = \left([X_2] \cup \Phi L_{H_2}\right)$, $[X_2]$ is complete ideal generated by $X_2 = \{\mu_{1C_1}y \mid y \in G_{12}, y = f_1x, x \in H_{12}\}$
(i) $\mu_{1G_{12}}: G_{12} \longrightarrow L_{G_2}$ is defined by $\mu_{1G_{12}}y = \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_{11}}}\Phi\mu_{1H_{12}}x\right)\right]$
(j) $\mu_{2G_2}: G_2 \longrightarrow L_{G_2}$ is defined by $\mu_{2G_2}y = \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_2}}\Phi\mu_{2H_2}x\right)\right]$

From definition of Fs-subsets $\mathcal{H}_1 \subseteq \mathcal{H}_2$ imply

(k) $H_{11} \subseteq H_{12} \Rightarrow f_1(H_{11}) \subseteq f_1(H_{12}) \Rightarrow G_{11} \subseteq G_{12}$,
$H_1 \supseteq H_2 \Rightarrow f_1(H_1) \supseteq f_1(H_2) \Rightarrow G_1 \supseteq G_2$
(l) $L_{H_1} \leq L_{H_2} \Rightarrow \Phi L_{H_1} \leq \Phi L_{H_2}, X_1 \subseteq X_2$
$\Rightarrow \Phi L_{H_1} \leq \Phi L_{H_2}, [X_1] \subseteq [X_2]$
$\Rightarrow \left([X_1] \cup \Phi L_{H_1}\right) \subseteq \left([X_2] \cup \Phi L_{H_2}\right) \Rightarrow L_{G_1} \leq L_{G_2}$
(m) $\mu_{1H_{11}}x \leq \mu_{1H_{12}}x, \forall x \in H_{11}$
$\Rightarrow \bigvee_{\substack{y=f_1x\\ x\in H_{11}}}\Phi\mu_{1H_{11}}x \leq \bigvee_{\substack{y=f_1x\\ x\in H_{12}}}\Phi\mu_{1H_{12}}x$
$\Rightarrow \mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_{11}}}\Phi\mu_{1H_{11}}x\right) \leq \mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_{12}}}\Phi\mu_{1H_{12}}x\right)$
$\Rightarrow \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_{11}}}\Phi\mu_{1H_{11}}x\right)\right] \leq \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_{12}}}\Phi\mu_{1H_{12}}x\right)\right]$
$\Rightarrow \mu_{1G_{11}}x \leq \mu_{1G_{12}}x$
And $\mu_{2H_1}x \geq \mu_{2H_2}x, \forall x \in H_2$
$\Rightarrow \bigvee_{\substack{y=f_1x\\ x\in H_1}}\Phi\mu_{2H_1}x \geq \bigvee_{\substack{y=f_1x\\ x\in H_2}}\Phi\mu_{2H_2}x$
$\Rightarrow \mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_1}}\Phi\mu_{2H_1}x\right) \geq= \circ\, \gamma\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_2}}\Phi\mu_{2H_2}x\right)$
$\Rightarrow \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_1}}\Phi\mu_{2H_1}x\right)\right] \geq \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_2}}\Phi\mu_{2H_2}x\right)\right]$

$\Rightarrow \mu_{2G_1}x \geq \mu_{2G_2}x$

(k),(l) and (m) imply $\mathcal{G}_1 \subseteq \mathcal{G}_2 \Rightarrow \bar{f}(\mathcal{H}_1) \subseteq \bar{f}(\mathcal{H}_2)$.

2.21 Image of Fs-empty set of first kind under an Fs-function:

Let $\Phi_{\mathcal{A}} = \mathcal{X} = \left(X_1, X, \bar{X}(\mu_{1X_1},\mu_{2X}), L_X\right)$, where

(1) $A \subseteq X_1 \cap X$ and $X_1 \not\supseteq X$ or

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(2) $\mu_{1D_1}x \neq \mu_{2D}x$,for $x \in X_1 \cap X$

We define $\bar{f}(\Phi_{\mathcal{A}}) = \Phi_{\mathcal{A}}$.

2.22 Result: $\bar{f}(\Phi_{\mathcal{A}}) = \Phi_{\mathcal{A}}$, where $\Phi_{\mathcal{A}} = \mathcal{D} = (D, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D)$,where $D_1 = D$ and $\Phi_{\mathcal{A}}$ is Fs-empty set of second

Proof: Suppose $\bar{f}(\Phi_{\mathcal{A}}) = \mathcal{E} = (E_1, E, \bar{E}(\mu_{1E_1}, \mu_{2E})L_E)$, where

(a) $E_1 = f_1(D) = E$
(b) $L_E = ([X] \cup \Phi L_D)$, [X] is complete ideal generated by $X = \{\mu_{1C_1}y | y \in E_1, y = f_1 x, x \in D_1 = D\}$
(c) $\mu_{1E_1}: E_1 \longrightarrow L_E$ is defined by

$$\mu_{1E_1}y = \mu_{2C} \vee \left[\mu_{1C_1} \wedge \left(\bigvee_{\substack{y=f_1x \\ x \in D_1 = D}} \Phi\mu_{1D_1}x\right)\right]$$
$$= \mu_{2C} \vee [\mu_{1C_1} \wedge \beta] \text{ , where}$$
$$\beta = \bigvee_{\substack{y=f_1x \\ x \in D_1 = D}} \Phi\mu_{1D_1}x$$

(d) $\mu_{2E}: E \longrightarrow L_E$ is defined by

$$\mu_{2E}y = \mu_{2C} \vee \left[\mu_{1C_1} \wedge \left(\bigvee_{\substack{y=f_1x \\ x \in D}} \Phi\mu_{2D}x\right)\right]$$
$$= \mu_{2C} \vee [\mu_{1C_1} \wedge \gamma] \text{ ,where}$$
$$\gamma = \bigvee_{\substack{y=f_1x \\ x \in D_1 = D}} \Phi\mu_{2D}x = \beta \ (\because \mu_{1D_1}x = \mu_{2D}x)$$

(d) and(e) imply $\mu_{1E_1}x = \mu_{2E}x = \alpha$,say

(e) $\bar{E}y = \mu_{1E_1}x \wedge (\mu_{2E}x)^c = \alpha \wedge (\alpha)^c = 0$

Hence $\bar{f}(\Phi_{\mathcal{A}}) = \Phi_{\mathcal{A}}$.

2.23 Proposition: For any Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$, $\bar{f}(\Phi_{\mathcal{A}}) = \Phi_{\mathcal{A}}$ where $\Phi_{\mathcal{A}}$ is Fs-empty set of first or Fs-empty set of second kind.

2.24 Proposition: For any Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$ and any two Fs-subsets $\mathcal{H}_1$ and $\mathcal{H}_2$ of $\mathcal{B}$, the following are true.

(1) $\bar{f}(\mathcal{H}_1 \cup \mathcal{H}_2) \supseteq \bar{f}(\mathcal{H}_1) \cup \bar{f}(\mathcal{H}_2)$
(2) $\bar{f}(\mathcal{H}_1 \cap \mathcal{H}_2) \subseteq \bar{f}(\mathcal{H}_1) \cap \bar{f}(\mathcal{H}_2)$

Proof:(1): $\mathcal{H}_1 \subseteq \mathcal{H}_1 \cup \mathcal{H}_2$ ($\because$ Proposition 1.10 in [4])

$\Rightarrow \bar{f}(\mathcal{H}_1) \subseteq \bar{f}(\mathcal{H}_1 \cup \mathcal{H}_2)$ ($\because$ Proposition 2.22) …..(I)

Similarly $\mathcal{H}_2 \subseteq \mathcal{H}_1 \cup \mathcal{H}_2$ ($\because$ Proposition 1.10 in [4])

$\Rightarrow \bar{f}(\mathcal{H}_2) \subseteq \bar{f}(\mathcal{H}_1 \cup \mathcal{H}_2)$ ($\because$ Proposition 2.22) …..(II)

(I) and (II) imply $\bar{f}(\mathcal{H}_1 \cup \mathcal{H}_2) \supseteq \bar{f}(\mathcal{H}_1) \cup \bar{f}(\mathcal{H}_2)$ ($\because$For a given family of Fs-subset $\mathcal{B}_i$and an Fs-set $\mathcal{C}$ such that $\mathcal{B}_i \subseteq \mathcal{C}$ for i∈I then$\bigcup_{i \in I} \mathcal{B}_i \subseteq \mathcal{C}$)

Proof:(2):Case(a): $\mathcal{H}_1 \cap \mathcal{H}_2 = \Phi_{\mathcal{A}} \Rightarrow \bar{f}(\mathcal{H}_1 \cap \mathcal{H}_2) = \bar{f}(\Phi_{\mathcal{A}}) = \Phi_{\mathcal{A}} \subseteq \bar{f}(\mathcal{H}_1) \cap \bar{f}(\mathcal{H}_2)$

Case (b): $\mathcal{H}_1 \cap \mathcal{H}_2 \subseteq \mathcal{H}_1$ ($\because$ Proposition 1.10)

$\Rightarrow \bar{f}(\mathcal{H}_1 \cap \mathcal{H}_2) \subseteq \bar{f}(\mathcal{H}_1)$ ($\because$ Proposition2.22) …..(III)

Similarly $\mathcal{H}_1 \cap \mathcal{H}_2 \subseteq \mathcal{H}_2$ ($\because$ Proposition 1.10 in [4])

$\Rightarrow \bar{f}(\mathcal{H}_1 \cap \mathcal{H}_2) \subseteq \bar{f}(\mathcal{H}_2)$ ($\because$ Proposition 2.22) …..(IV)

(III) and (IV) imply $\bar{f}(\mathcal{H}_1 \cap \mathcal{H}_2) \subseteq \bar{f}(\mathcal{H}_1) \cap \bar{f}(\mathcal{H}_2)$ ($\because$ Proposition 1.14.1 in [4])

2.25 Proposition: For any Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$ and any family of Fs-subsets $\mathcal{H}_i$, i∈I of $\mathcal{B}$ the following are true.

(a) $\bar{f}(\bigcup_{i \in I} \mathcal{H}_i) \supseteq \bigcup_{i \in I} \bar{f}(\mathcal{H}_i)$
(b) $\bar{f}(\bigcap_{i \in I} \mathcal{H}_i) \subseteq \bigcap_{i \in I} \bar{f}(\mathcal{H}_i)$

Proof:(a): $\mathcal{H}_i \subseteq \bigcup_{i \in I} \mathcal{H}_i$ ($\because$ Proposition 1.13 in [4])

$\Rightarrow \bar{f}(\mathcal{H}_i) \subseteq \bar{f}(\bigcup_{i \in I} \mathcal{H}_i)$ ($\because$ Proposition 2.22)

$\bar{f}(\bigcup_{i \in I} \mathcal{H}_i) \supseteq \bigcup_{i \in I} \bar{f}(\mathcal{H}_i)$ ($\because$For a given family of Fs-subset $\mathcal{B}_i$and an Fs-set $\mathcal{C}$ such that $\mathcal{B}_i \subseteq \mathcal{C}$ for i∈I then$\bigcup_{i \in I} \mathcal{B}_i \subseteq \mathcal{C}$)

The proof of (b): The proof follows clearly

2.26 Result: If $\bar{f}$ is increasing Fs-function, $\mathcal{D} \subseteq \mathcal{B}$ and $\bar{f}_i(\mathcal{D}) = \mathcal{E} = (E_1, E, \bar{E}(\mu_{1E_1}, \mu_{2E}), L_E)$ then $\mu_{1E_1}y = \bigvee_{\substack{y=f_1x \\ x \in D_1}} \Phi\mu_{1D_1}x$ and $\mu_{2E}y = \bigvee_{\substack{y=f_1x \\ x \in D}} \Phi\mu_{2D}x$.

Proof: Given $\bar{f}(\mathcal{D}) = \mathcal{E} = (E_1, E, \bar{E}(\mu_{1E_1}, \mu_{2E})L_E)$, where

(a) $E_1 = f_1(D_1)$
(b) $E = f_1(D)$
(c) $L_E = ([X] \cup \Phi L_D)$, [X] is complete ideal generated by $X = \{\mu_{1C_1}y | y \in E_1, y = f_1 x, x \in D_1\}$
(d) $\mu_{1E_1}: E_1 \longrightarrow L_E$ is define by

$$\mu_{1E_1}y = \mu_{2C} \vee \left[\mu_{1C_1} \wedge \left(\bigvee_{\substack{y=f_1x \\ x \in D_1}} \Phi\mu_{1D_1}x\right)\right] \text{ given}$$

$\bar{f} = \bar{f}_i$.

For $x \in D_1$, $\mu_{1D_1}x \leq \mu_{1B_1}x$ and Φ is a complete homomorphism imply

$\Phi\mu_{1D_1}x \leq \Phi\mu_{1B_1}x \leq (\mu_{1C_1} \circ f_1)x = \mu_{1C_1}y$ inturn imply

$\bigvee_{\substack{y=f_1x \\ x \in D_1}} \Phi\mu_{1D_1}x \leq \mu_{1C_1}y$ . . . .(I)

Again, $\mu_{1D_1}x \geq \mu_{2D}x \geq \mu_{2B}x$, for each $x \in D_1$ inturn imply

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$\Phi\mu_{1D_1}x \geq \Phi\mu_{2D}x \geq \Phi\mu_{2B}x \geq (\mu_{2C}\circ f)x = (\mu_{2C}\circ f_1)x = \mu_{2C}y$ and

$\bigvee_{\substack{y=f_1x\\ x\in D_1}} \Phi\mu_{1D_1}x \geq \mu_{2C}y$ . . . . . .(II)

Therefore from(I) and(II) we get $\mu_{1E_1}y = \bigvee_{\substack{y=f_1x\\ x\in D_1}} \Phi\mu_{1D_1}x$

(e) $\mu_{2E}: E \longrightarrow L_E$ is define by

$$\mu_{2E}y = \mu_{2C}\vee\left[\mu_{1C_1}\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in D}} \Phi\mu_{2D}x\right)\right]$$

for $x\in B$, $\mu_{2D}x \geq \mu_{2B}x$ imply

$\Phi\mu_{2D}x \geq \Phi\mu_{2B}x \geq (\mu_{2C}\circ f)x = (\mu_{2C}\circ f_1)x = \mu_{2C}y$ inturn imply

$\bigvee_{\substack{y=f_1x\\ x\in D}} \Phi\mu_{2D}x \geq \mu_{2C}y$ . . . . . .(III)

Again, $\Phi\mu_{2D}x \leq \Phi\mu_{1D_1}x \leq \Phi\mu_{1B_1}x \leq (\mu_{1C_1}\circ f_1)x = \mu_{1C_1}y$ inturn imply

$\bigvee_{\substack{y=f_1x\\ x\in D}} \Phi\mu_{2D}x \leq \mu_{1C_1}y$ . . . . . .(IV)

Therefore from(III) and(IV) we get $\mu_{2E}y = \bigvee_{\substack{y=f_1x\\ x\in D}} \Phi\mu_{2D}x$

2.27 Result: If $\bar{f}$ is decreasing Fs-function, $\mathcal{D}\subseteq\mathcal{B}$ and $\bar{f}_i(\mathcal{D}) = \mathcal{E} = (E_1, E, \bar{E}(\mu_{1E_1}, \mu_{2E}), L_E)$ then

$$\mu_{1E_1}y = \mu_{2C}\vee\left[\mu_{1C_1}\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in D_1}} \Phi\mu_{1D_1}x\right)\right]$$

and $\mu_{2E}y = \mu_{2C}\vee\left[\mu_{1C_1}\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in D}} \Phi\mu_{2D}x\right)\right]$

2.28 Result: If $\bar{f}$ is preserving Fs-function, $\mathcal{D}\subseteq\mathcal{B}$ and $\bar{f}_p(\mathcal{D}) = \mathcal{E} = (E_1, E, \bar{E}(\mu_{1E_1}, \mu_{2E})L_E)$ then $\mu_{1E_1}y = \bigvee_{\substack{y=f_1x\\ x\in D_1}} \Phi\mu_{1D_1}x$ and $\mu_{2E}y = \bigvee_{\substack{y=f_1x\\ x\in D}} \Phi\mu_{2D}x$.

Proof: Given $\bar{f}(\mathcal{D}) = \mathcal{E} = (E_1, E, \bar{E}(\mu_{1E_1}, \mu_{2E})L_E)$, where

(a) $E_1 = f_1(D_1)$

(b) $E = f_1(D)$

(c) $L_E = ([X]\cup\Phi L_D)$, $[X]$ is complete ideal generated by $X = \{\mu_{1C_1}y | y\in E_1, y = f_1x, x\in D_1\}$

(d) $\mu_{1E_1}: E_1 \longrightarrow L_E$ is defined by $\mu_{1E_1}y = \mu_{2C}\vee\left[\mu_{1C_1}\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in D_1}} \Phi\mu_{1D_1}x\right)\right]$ given $\bar{f} = \bar{f}_i$.

For $x\in D_1$, $\mu_{1D_1}x \leq \mu_{1B_1}x$ and Φ is a complete homomorphism imply

$\Phi\mu_{1D_1}x \leq \Phi\mu_{1B_1}x = (\mu_{1C_1}\circ f_1)x = \mu_{1C_1}y$ inturn imply

$\bigvee_{\substack{y=f_1x\\ x\in D_1}} \Phi\mu_{1D_1}x \leq \mu_{1C_1}y$ . . . .(I)

Again, $\mu_{1D_1}x \geq \mu_{2D}x \geq \mu_{2B}x$, for each $x\in D_1$ inturn imply

$\Phi\mu_{1D_1}x \geq \Phi\mu_{2D}x \geq \Phi\mu_{2B}x = (\mu_{2C}\circ f)x = (\mu_{2C}\circ f_1)x = \mu_{2C}y$ and

$\bigvee_{\substack{y=f_1x\\ x\in D_1}} \Phi\mu_{1D_1}x \geq \mu_{2C}y$ . . . . . .(II)

Therefore from(I) and(II) we get $\mu_{1E_1}y = \bigvee_{\substack{y=f_1x\\ x\in D_1}} \Phi\mu_{1D_1}x$

(e) $\mu_{2E}: E \longrightarrow L_E$ is define by

$$\mu_{2E}y = \mu_{2C}\vee\left[\mu_{1C_1}\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in D}} \Phi\mu_{2D}x\right)\right]$$

for $x\in B$, $\mu_{2D}x \geq \mu_{2B}x$ imply

$\Phi\mu_{2D}x \geq \Phi\mu_{2B}x = (\mu_{2C}\circ f)x = (\mu_{2C}\circ f_1)x = \mu_{2C}y$ inturn imply

$\bigvee_{\substack{y=f_1x\\ x\in D}} \Phi\mu_{2D}x \geq \mu_{2C}y$ . . . . . .(III)

Again, $\Phi\mu_{2D}x \leq \Phi\mu_{1D_1}x \leq \Phi\mu_{1B_1}x = (\mu_{1C_1}\circ f_1)x = \mu_{1C_1}y$ inturn imply

$\bigvee_{\substack{y=f_1x\\ x\in D}} \Phi\mu_{2D}x \leq \mu_{1C_1}y$ . . . . . .(IV)

Therefore from(III) and(IV) we get $\mu_{2E}y = \bigvee_{\substack{y=f_1x\\ x\in D}} \Phi\mu_{2D}x$.

2.29 Proposition: For any pair of Fs-functions $\bar{f}: \mathcal{B}\to\mathcal{C}$ and $\bar{g}: \mathcal{C}\to\mathcal{D}$ and any Fs-subset $\mathcal{H}$ of $\mathcal{B}$ the following is true

$$(\bar{g}\circ\bar{f})(\mathcal{H}) \subseteq \bar{g}\left(\bar{f}(\mathcal{H})\right)$$

Proof: LHS: $(\bar{g}\circ\bar{f})(\mathcal{H}) = [g_1\circ f_1, g\circ f, \Psi\circ\Phi](\mathcal{H}) = \mathcal{G} = (G_1, G, \bar{G}(\mu_{1G_1}, \mu_{2G}), L_G)$ say

(a) $G_1 = (g_1\circ f_1)(H_1)$

(b) $G = (g_1\circ f_1)(H)$

(c) $L_G = ([X]\cup\Phi L_H)$, $[X]$ is complete ideal generated by $X = \{\mu_{1D_1}z | z\in G_1, z = (g_1\circ f_1)x, x\in H_1\}$

(d) $\mu_{1G_1}: G_1 \longrightarrow L_G$ is defined by $\mu_{1G_1}z = \mu_{2D}z\vee\left[\mu_{1D_1}z\wedge\left(\bigvee_{\substack{z=(g_1\circ f_1)x\\ x\in H_1}} \Psi\Phi\mu_{1H_1}x\right)\right]$

(e) $\mu_{2G}: G \longrightarrow L_G$ is defined by

$$\mu_{2G}z = \mu_{2D}z\vee\left[\mu_{1D_1}\wedge\left(\bigvee_{\substack{z=(g_1\circ f_1)x\\ x\in H}} \Psi\Phi\mu_{2H}x\right)\right]$$

Let $\bar{f}(\mathcal{H}) = \mathcal{K} = (K_1, K, \bar{K}(\mu_{1K_1}, \mu_{2K}), L_K)$, where

(f) $K_1 = f_1(H_1)$

(g) $K = f_1(H)$

(h) $L_K = ([X_1]\cup\Phi L_H)$, $[X_1]$ is complete ideal generated by $X_1 = \{\mu_{1C_1}y | y\in K_1, y = f_1x, x\in H_1\}$,

(i) $\mu_{1K_1}: K_1 \longrightarrow L_K$ is defined by $\mu_{1K_1}y = \mu_{2C}\vee\left[\mu_{1C_1}\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H_1}} \Phi\mu_{1H_1}x\right)\right]$

(j) $\mu_{2K}: K \longrightarrow L_K$ is defined by

$$\mu_{2K}y = \mu_{2C}\vee\left[\mu_{1C_1}\wedge\left(\bigvee_{\substack{y=f_1x\\ x\in H}} \Phi\mu_{2H}x\right)\right]$$

RHS: $\bar{g}\left(\bar{f}(\mathcal{H})\right) = \bar{g}(\mathcal{K}) = \mathcal{M} = (M_1, M, \bar{M}(\mu_{1M_1}, \mu_{2M}), L_M)$ say

(k) $M_1 = g_1(K_1) = g_1(f_1(H_1)) = (g_1 \circ f_1)(H_1)$

(l) $M = g_1(K) = g_1(f_1(H)) = (g_1 \circ f_1)(H)$

(m) $L_M = ([X_2] \cup \Psi L_K)$, $[X_2]$ is complete ideal generated by $X_2 = \{\mu_{1D_1} z | z \in M_1 = G_1, z = g_1 y, y \in K_1\}$

(n) $\mu_{1M_1} : M_1 \longrightarrow L_M$ is defined by $\mu_{1M_1} z = \mu_{2D} z \vee \left[\mu_{1D_1} z \wedge \left(\vee_{\substack{z=g_1 y \\ y \in K_1}} \Psi \mu_{1K_1} y\right)\right]$

(o) $\mu_{2M} : M \longrightarrow L_M$ is defined by $\mu_{2M} z = \mu_{2D} z \vee \left[\mu_{1D_1} \wedge \left(\vee_{\substack{z=g_1 y \\ y \in K}} \Psi \mu_{2K} y\right)\right]$

Clearly

(p) $G_1 = M_1$ follows from (a) and(k)

(q) $G = M$ follows from (b) and(l)

(r) L_G is a complete subalgebra of L_M i.e. $L_G \leq L_M$ follows from (c) and(m)

(s) $\mu_{1G_1} z \leq \mu_{1M_1} z$, for each $z \in G_1 = M_1$ follows from (d) and(n)

(t) $\mu_{2G} z \geq \mu_{2M} z$, for each $z \in G = M$ follows from (e) and(m)

From all the above statements we can easily conclude that

$$(\bar{g} \circ \bar{f})(\mathcal{H}) \subseteq \bar{g}\left(\bar{f}(\mathcal{H})\right).$$

CONCLUSION

We can observe that similarities between results in theory of Fs-functions and some results in the theory of crisp functions .

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