

# Designing Multi-User MIMO for Energy and Spectral Efficiency



G.Ramya<sup>1</sup>, S.Pedda Krishna.<sup>2</sup>, Dr.M.Narsing Yadav<sup>3</sup>

1.PG. Student, MRIET, Hyderabad, AP,INDIA, ramyagujjula275@gmail.com

2. Assistant Professor, MRIET, Hyderabad, AP,INDIA, krishna.samineni@gmail.com

3. Professor,HOD, Department of ECE,MRIET, Hyderabad, AP, INDIA,narsing.mriet@gmail.com

**Abstract:-** Multi-user multiple-input multiple output(MIMO) communication system must be designed to cover a given area with maximal energy efficiency (bits/Joule). A multiplicity of autonomous terminal simultaneously transmits data stream to a compact array of antennas. A array uses imperfect channel-state information derived from transmitted pilots to extract the individual data streams. The power radiated by the terminals can be made inversely proportional to the square-root of the number of base station antennas with no reduction in performance. In contrast if perfect channel-state information were available the power could be made inversely proportional to the number of antennas. Lower capacity bounds for maximum-ratio combining (MRC) and zero-forcing (ZF) detection are derived. A MRC receiver normally performs worse than ZF and MMSE. However as power levels are reduced, the cross-talk introduced by the inferior maximum-ratio receiver eventually falls below the noise level and this simple receiver becomes a viable option. The tradeoff between the energy efficiency (bits/J) and spectral efficiency (bits/channel use/terminal) is quantified.

**Keywords:** Energy efficiency, Spectral efficiency, Multiuser MIMO.

## I INTRODUCTION

The design of current wireless networks (e.g., based on the Long-Term Evolution (LTE) standard) have been mainly driven by enabling high spectral efficiency due to the spectrum shortage and rapidly increasing demand for data services [1]. As a result, these networks are characterized by poor energy efficiency (EE) and large disparity between peak and average rates. The former is defined as the number of bits transferred per Joule of energy and it is affected by many factors such as (just to name a few) network architecture, spectral efficiency, radiated transmit power, and circuit power consumption [1]–[3]. Motivated by environmental and economical costs, green radio is a new research direction that aims at designing wireless networks with better coverage and higher EE [2]. This paper analyzes the potential for power savings on the uplink of MU-MIMO systems. We derive new capacity bounds of the uplink for finite number of BS antennas. These results are different from recent results in [4] and [5]. In [4] and [5], the authors derived a deterministic equivalent of the SINR assuming that the number of transmit antennas and the number of users go to infinity but their ratio remains bounded for the downlink of network MIMO systems using a sophisticated scheduling scheme and MISO broadcast channels using zero-forcing (ZF) pre coding, respectively.

While it is well known that MIMO technology can offer improved power efficiency, owing to both array gains and diversity effects [3]. We study the tradeoff between spectral efficiency and energy efficiency. For imperfect CSI, in the low transmit power regime, we can simultaneously increase the spectral-efficiency and energy-efficiency. We further show that in MU-MIMO, very high spectral efficiency can be obtained even with simple MRC processing at the same time as the transmit power can be cut back by orders of magnitude and that this holds true even when taking into account the losses associated with acquiring CSI from uplink pilots. MRC also has the advantage that it can be implemented in a distributed manner, i.e., each antenna performs multiplication of the received signals with the conjugate of the channel, without sending the entire baseband signal to the BS for processing.

## II SYSTEM MODEL

We consider the uplink of a MU-MIMO system. The system includes one BS equipped with an array of  $M$  antennas that receive data from  $K$  single-antenna users. The nice thing about single-antenna users is that they are inexpensive, simple, and power-efficient, and each user still gets typically high throughput. Furthermore, the assumption that users have single antennas can be considered as a special case of users having multiple antennas when we treat the extra antennas as if they were additional autonomous users. The users transmit their data in the same time-frequency resource. The  $M \times 1$  received vector at the BS is

$$y = \sqrt{p_u} Hx + n \quad (2.1)$$

Where  $H$  represents the  $M \times K$  channel matrix between the BS and the  $K$  users,  $\sqrt{p_u}x$  is the  $K \times 1$  vector of symbols simultaneously transmitted by the  $K$  users (the average transmitted power of each user is  $p_u$ ); and  $n$  is a vector of additive white, zero-mean Gaussian noise. We take the noise variance to be 1, to minimize notation, but without loss of generality. For favorable propagations consider an  $M \times K$  uplink (multiple-access) MIMO channel  $H$ , where  $M \geq K$ , neglecting for now path loss. This channel can offer a sum-rate of

$$R = \sum_{k=1}^K \log_2(1 + p_u \lambda_k^2) \quad (2.2)$$

Where  $p_u$  is the power spent per terminal and  $\{\lambda_k\}_{k=1}^K$  are the singular values of  $H$ .

### III ACHIEVABLE RATE

By using a large antenna array, we can reduce the transmitted power of the users as  $M$  grows large, while maintaining a given, desired quality-of-service. In this section, we quantify this potential for power decrease, and derive achievable rates of the uplink. Theoretically, the BS can use the maximum-likelihood detector to obtain optimal performance. However, the complexity of this detector grows exponentially with  $K$ . The interesting operating regime is when both  $M$  and  $K$  are large, but  $M$  is still (much) larger than  $K$ , i.e.,  $1 \ll K \ll M$ . It is known that in this case, linear detectors (MRC, ZF and MMSE) perform fairly well [8] and therefore we will restrict consideration to those detectors in this paper. We treat the cases of perfect CSI and estimated CSI separately.

#### Perfect Channel State Information

I.

We first consider the case when the BS has perfect CSI, i.e. it knows  $H$ . Let  $A$  be an  $M \times K$  linear detector matrix which depends on the channel  $H$ . By using the linear detector, the received signal is separated into streams by multiplying it with  $A^H$  as follows

$$r = A^H y \quad (3.1)$$

We consider three conventional linear detectors MRC, ZF, and MMSE, i.e.,

$$A = H \quad \text{for MRC}$$

$$= H(H^H H)^{-1} \quad \text{for ZF}$$

$$= H \left( H^H H + \frac{1}{P_u} I_K \right)^{-1} \quad \text{for MMSE} \quad (3.2)$$

From (2.1) and (3.1), the received vector after using the linear detector is given by

$$r = \sqrt{P_u} A^H H x + A^H n \quad (3.3)$$

Where  $r_k$  and  $x_k$  be the  $k^{\text{th}}$ , element of the  $K \times 1$  vectors  $r$  and  $x$ , respectively. Then

$$\begin{aligned} r_k &= \sqrt{P_u} a_k^H H x + a_k^H n \\ &= \sqrt{P_u} a_k^H h_k x_k + \sqrt{P_u} \sum_{i=1, i \neq k}^K a_k^H h_i x_i + a_k^H n \end{aligned} \quad (3.4)$$

Where  $a_k$  and  $h_k$  are the  $k^{\text{th}}$  columns of the matrices  $A$  and  $H$  respectively. For a fixed channel realization  $H$ , the noise-plus-interference term is a random variable with zero mean and variance

$$P_u \sum_{i=1, i \neq k}^K |a_k^H h_i|^2 + \|a_k\|^2 \quad (3.5)$$

Assuming further that the channel is ergodic so that each code word span over a large (infinity) number of realizations of fast fading of  $H$ , the ergodic achievable uplink rate of the  $k^{\text{th}}$  user is

$$R_{p,k} = E \left\{ \log_2 \left( 1 + \frac{P_u |a_k^H h_k|^2}{P_u \sum_{i=1, i \neq k}^K |a_k^H h_i|^2 + \|a_k\|^2} \right) \right\} \quad (3.6)$$

**Case 1:** Assume that the BS has perfect CSI and that the transmit power of each user is scaled with  $M$  according to  $P_u = E_u/M$ ,  $E_u$  is fixed. Then

$$R_{p,k} \rightarrow \log_2(1 + \beta_k E_u), \quad M \rightarrow \text{infinity} \quad (3.7)$$

**a) Maximum-Ratio Combining:** With MRC,  $A=H$  so

$a_k = h_k$  From (8), the achievable uplink rate of  $k^{\text{th}}$  user is:

$$R_{p,k}^{\text{MRC}} = E \left\{ \log_2 \left( 1 + \frac{P_u \|h_k\|^4}{P_u \sum_{i=1, i \neq k}^K |h_k^H h_i|^2 + \|h_k\|^2} \right) \right\} \quad (3.8)$$

**b) Zero Forcing Receiver:** With ZF,  $A^H H = I_K$

therefore,  $a_k^H h_i = \delta_{k,i}$  where  $\delta_{k,i} = 1$  where  $k=i$  and 0 otherwise. From (7) the uplink rate of the  $k^{\text{th}}$  user is:

$$R_{p,k}^{\text{ZF}} = E \left\{ \log_2 \left( 1 + \frac{P_u}{[(H^H H)^{-1}]_{k,k}} \right) \right\} \quad (3.9)$$

### IV. ENERGY-EFFICIENCY VERSUS SPECTRAL-EFFICIENCY TRADEOFF

The energy-efficiency (in bits/Joule) of a system is defined as the spectral-efficiency (sum-rate in bits/channel use) divided by the transmit power expended (in Joules/channel use). Typically, increasing the spectral efficiency is associated with increasing the power and hence, with decreasing the energy efficiency. Therefore, there is a fundamental tradeoff between the energy efficiency and the spectral efficiency. However, in one operating regime it is possible to jointly increase the energy and spectral efficiencies, and in this regime there is no tradeoff.

In this section, we study the energy-spectral efficiency tradeoff for the uplink of MU-MIMO systems using linear receivers at the BS. Certain activities (multiplexing to many users rather than beam forming to a single user and increasing the number of service antennas) can simultaneously benefit both the spectral-efficiency and the radiated energy-efficiency. Once the number of service antennas is set, one can adjust other system parameters (radiated power, numbers of users, duration of pilot sequences) to obtain increased spectral-efficiency at the cost of reduced energy-efficiency, and vice-versa. This should be a desirable feature for service providers: they can set the operating point according to the current traffic demand (high energy-efficiency and low spectral-efficiency, for example, during periods of low demand).

**Single-Cell MU-MIMO Systems:** We define the spectral efficiency for perfect and imperfect CSI, respectively, as follows

$$R_P^A = \sum_{k=1}^K R_{P,k}^A \quad \& \quad R_{IP}^A = \frac{T-T}{T} \sum_{k=1}^K R_{IP,k}^A \quad (4.1)$$

Where  $A \in \{MRC, ZF, MMSE\}$  corresponds to MRC, ZF and MMSE, and  $T$  is coherence interval in symbols. The energy efficiency for perfect and imperfect CSI is defined as:

$$\eta_P^A = \frac{1}{p_u} R_P^A \quad \& \quad \eta_{IP}^A = \frac{1}{p_u} R_{IP}^A \quad (4.2)$$

For perfect CSI, it is straightforward to that when the spectral efficiency increases, the energy efficiency decreases. For imperfect CSI, this is not always so.

**a) Maximum- Ratio combining:** The spectral efficiency and energy efficiency with MRC processing are given by

$$R_{IP}^{MRC} = \frac{T-\tau}{T} K \log_2 \left( 1 + \frac{\tau(M-1)p_u^2}{\tau(K-1)p_u^2 + (K+\tau)p_u + 1} \right) \quad \& \quad \eta_{IP}^{MRC} = \frac{1}{p_u} R_{IP}^{MRC} \quad (4.3)$$

For low  $p_u$  the energy efficiency increases when  $p_u$  increases, and for high  $p_u$  the energy efficiency decreases when  $p_u$  increases. The relation between the spectral efficiency and energy efficiency at  $p_u \ll 1$ :

$$\eta_{IP}^{MRC} \approx \sqrt{\frac{T-\tau}{T}} K \cdot \log_2(e) \tau (M-1) \cdot R_{IP}^{MRC} \quad (4.4)$$

We can see that when  $p_u \ll 1$  by doubling the spectral efficiency, or by doubling  $M$ , we can increase the energy efficiency by 1.5 dB.

**Zero-Forcing Receiver:** The spectral efficiency and energy efficiency for ZF are given by

$$R_{IP}^{ZF} = \frac{T-\tau}{T} K \log_2 \left( 1 + \frac{\tau(M-K)p_u^2}{(K+\tau)p_u + 1} \right) \quad \& \quad \eta_{IP}^{ZF} = \frac{1}{p_u} R_{IP}^{ZF} \quad (4.5)$$

Similarly to in the analysis of MRC, we can show that at low transmit power  $p_u$ , the energy efficiency increases when the spectral efficiency increases. In the low- $p_u$  regime, we obtain the following:

$$\eta_{IP}^{ZF} \approx \sqrt{\frac{T-\tau}{T}} K \cdot \log_2(e) \tau (M-K) \cdot R_{IP}^{ZF} \quad (4.6)$$

Again, at  $p_u \ll 1$  by doubling  $M$  or  $R_{IP}^{ZF}$ , we can increase the energy efficiency by 1.5 dB.

## V. NUMERICAL RESULTS

We assume that the transmitted data are modulated with OFDM. Here, we choose parameters that resemble those of LTE standard: OFDM symbol duration of  $T_s = 71.4 \mu s$  and useful symbol duration of  $T_u = 66.7 \mu s$ . Therefore, the guard interval length is  $T_g = T_s - T_u = 4.7 \mu s$ . We choose the channel coherence time to be  $T_c = 1$  ms. Then,

$$T = \frac{T_c}{T_s}, \frac{T_u}{T_g} = 196,$$

where  $\frac{T_c}{T_s} = 14$  is the number of OFDM symbols in a 1 ms

coherence interval, and  $\frac{T_c}{T_g} = 14$  corresponds to the “frequency smoothness interval”.

### Energy Efficiency versus Spectral Efficiency Tradeoff:

We examine the tradeoff between energy efficiency and spectral efficiency in more detail. Here, we ignore the effect of large-scale fading, i.e., we set  $D = IK$ . We normalize the energy efficiency against a reference mode corresponding to a single-antenna BS serving one single-antenna user with  $p_u = 10$  dB. For this reference mode, the spectral efficiencies and energy efficiencies for MRC, ZF, and MMSE are equal, and given by

$$R_{IP}^0 = \frac{T-\tau}{T} E \left\{ \log_2 \left( 1 + \frac{\tau p_u^2 |Z|^2}{1 + p_u(1+\tau)} \right) \right\} \quad \& \quad \eta_{IP}^0 = \frac{R_{IP}^0}{p_u} \quad (5.1)$$

Fig. 1 shows the relative energy efficiency versus the spectral efficiency for MRC and ZF. The relative energy efficiency is obtained by normalizing the energy efficiency by  $\eta_{IP}^0$  and it is therefore dimensionless. The dotted and dashed lines show the performances for the cases of  $M = 1, K = 1$  and  $M = 100, K = 1$ , respectively. Each point on the curves is obtained by choosing the transmit power  $p_u$  and pilot sequence length  $\tau$  to maximize the energy efficiency for a given spectral efficiency. The solid lines show the performance for the cases of  $M = 50$ , and 100. Each point on these curves is computed by jointly choosing  $K, \tau$  and  $p_u$  to maximize the energy-efficiency subject a fixed spectral-efficiency. We next consider a multiuser system ( $K > 1$ ). Here the transmit power  $p_u$ , the number of users  $K$ , and the duration of pilot sequences  $\tau$  are chosen optimally for fixed  $M$ . We consider  $M = 50$  and 100. Here the system performance improves very significantly compared to the single-user case. For example, with MRC, at  $p_u = 0$  dB, compared with the case of  $M = 1, K = 1$ , the spectral-efficiency increases by factors of 50 and 80, while the energy-efficiency increases by factors of 55 and 75 for  $M = 50$  and  $M = 100$ , respectively.

The corresponding optimum values of  $K$  and  $\tau$  as functions of the spectral efficiency for  $M = 100$  are shown in Fig. 2. For MRC, the optimal number of users and uplink pilots are the same (this means that the minimal possible lengths of training sequences are used). For ZF, more of the coherence interval is used for training. Generally, at low transmit power and therefore at low spectral efficiency, we spend more time on training than on payload data transmission. At high power (high spectral efficiency and low energy efficiency), we can serve around 55 users, and  $K = \tau$  for both MRC and ZF.

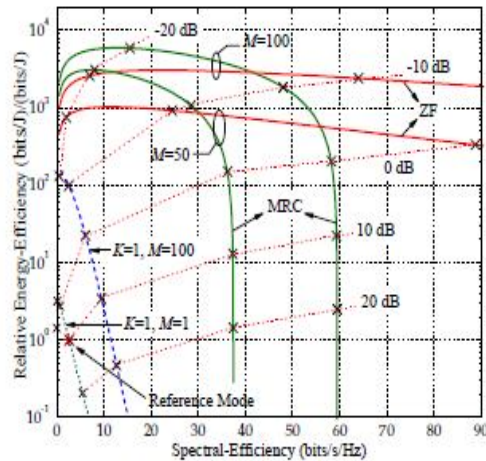


Fig. 1. Energy efficiency (normalized with respect to the reference mode) versus spectral efficiency for MRC and ZF receiver processing with imperfect CSI.

The reference mode corresponds to  $K = 1, M = 1$  (single antenna, single user), and a transmit power of  $p_u = 10$  dB. The coherence interval is  $T = 196$  symbols. For the dashed curves (marked with  $K = 1$ ), the transmit power  $p_u$  and the fraction of the coherence interval  $\tau/T$  spent on training was optimized in order to maximize the energy efficiency for a fixed spectral efficiency. For the green and red curves (marked MRC and ZF; shown for  $M = 50$  and  $M = 100$  antennas, respectively), the number of users  $K$  was optimized jointly with  $p_u$  and  $\tau/T$  to maximize the energy efficiency for given spectral efficiency. Any operating point on the curves can be obtained by appropriately selecting  $p_u$  and optimizing with respect to  $K$  and  $\tau/T$ . The number marked next to the  $\times$  marks on each curve is the power  $p_u$  spent by the transmit.

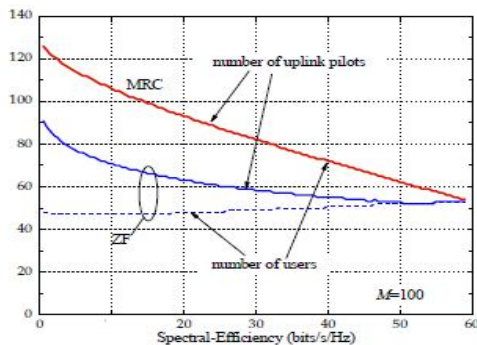


Fig. 2. Optimal number of users  $K$  and number of symbols  $\tau$  spent on training, out of a total of  $T = 196$  symbols per coherence interval, for the curves in Fig. 6 corresponding to  $M = 100$  antennas.

## VI. CONCLUSION

Multuser MIMO systems offer the opportunity of increasing the spectral efficiency (in terms of bits/s/Hz sum-rate in a given cell) by one or two orders of magnitude, and simultaneously improving the energy efficiency (in terms of

bits/J) by three orders of magnitude. This is possible with simple linear processing such as MRC or ZF at the BS, and using channel estimates obtained from uplink pilots even in a high mobility environment where half of the channel coherence interval is used for training. Generally, ZF outperforms MRC owing to its ability to cancel intra cell interference. However, in multi cell environments with strong pilot contamination, this advantage tends to diminish. MRC has the additional benefit of facilitating a distributed per-antenna implementation of the detector. These conclusions are valid in an operating regime where 100 antennas serve about 50 terminals in the same time-frequency resource, each terminal having a fading-free throughput of about 1 bpcu, and hence the system offering a sum-throughput of about 50 bpcu.

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