ISSN 2278 - 3091

International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE), Vol.2 , No.5, Pages : 01-08 (2013)
Special Issue of ICACET 2013 - Held during October 14-15, 2013, Kuala Lumpur, Malaysia

Review of Image Denoising Algorithms Based on the Wavelet Transformation

AsemKhmag[1], AbdRahmanRamli[2],ShaifulJahari Hashim[3], Syed Abdul Rahman Al-Haddad[4]
[2,3,4]Department of Computer and Communication Systems Engineering, Universiti Putra Malaysia, Malaysia,
[1]khmaj2002@gmail.com
[2]arr@eng.upm.edu.my
[3]sar@eng.upm.edu.my
[4]shaiful@eng.upm.edu.my

Abstract *:* The search for efficient image denoising methods is still a valid challenge at the crossing of functional analysis and statistics. In spite of the sophistication of the recently methods, most algorithms have not yet attained a desirable level of applicability. All the algorithms show a high outstanding Performance when the image model corresponds to the algorithm assumptions but it fails in general and create artifacts or change the main structures of the original image. De-noising of natural images corrupted by white Gaussian noise using wavelet techniques is very effective because of its ability to capture the energy of the signal in few energy transform values or coefficients. This method performs well under a number of applications because wavelet transform has the compaction property of having only a small number of large coefficients where the remaining wavelet coefficients are very small. The aim of this review paper is to examine all existing studies in the literature related to applying wavelet transformation for denoising images. However, to review various denoising algorithms using wavelet transform; those algorithms are discussed and showed how the appearance and quality of the noisy image can be improved. Algorithms such as SUREShrink, VisuShrink,BayesShrink, Bivariate shrink, Neigh Shrink and Normal shrink are presented in this paper. In the part of the experimental results, different Gaussian white noise levels in PSNR are shown.

Key words : Denoising, discrete wavelet transforms (DWT), hard and soft thresholding and peak signal to noise ratio (PSNR).

INTRODUCTION

The need for efficient image restoration methods has grown with the massive production in the field of digital images and movies, often taken in poor conditions. No matter how good cameras are, an image improvement is always desirable to extend their range of actions. Visual information transmitted in the form of digital images is becoming a major method of communication in the modern technology, unfortunately; the image obtained after transmission is often corrupted with different kinds of noise [1].

The received images need processing before it can be used in several applications. Image denoising involves the manipulation of the image data to produce a visually high quality image. Over the past decade, wavelet transforms have received a lot of attention from researchers in many different areas. The discrete wavelet transform also provides multiscale spatial and frequency decomposition. The frequencies can be resolved in space and this is very useful for locating particular features of interest in an image. For this reason, it is preferred over other methods such as the Fourier transform, Gaussian lowpass filter and Wiener filter. This paper reviews wavelet based approach, where the additive noise form is present in an image, Gaussian noise is most commonly known as additive white Gaussian noise which is evenly distributed over the signal. Each pixel in the noisy image is the sum of the true pixel value and random Gaussian distributed noise value [2]. Simple denoising algorithms that use the wavelet transform consist of three steps. [3] Proposed the following wavelet denoising scheme:

- Calculate the wavelet transform (w) of the noisy signal(y).
- Modify the noisy wavelet coefficients according to threshold function *T(.)* (Hard or Soft) to obtain coefficients $\tilde{\theta}$
- Compute the inverse transform using the modified coefficients to collect the original signal $\tilde{x}$

Fig. 1 shows block diagram of Donoho algorithm [4].

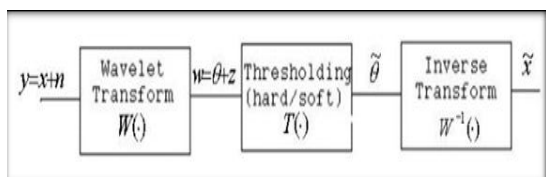


Fig.1. The Basic Framework of the Wavelet Transform Based Image Denoising

Bui and Chen [5] proposed a translation invariant multiwavelet denoising scheme that gave better results than [6]. Simoncelli and Adelson [7] propose Bayesian wavelet coring method to reduce the visual artifacts: Gibbs phenomena in the neighborhood of discontinuities. Spurious wavelets can also be seen in the restored image due to the cancelation of small coefficients; this artifact will be called wavelet outliers, as it is introduced in [8].Coifman and Donoho [9] improved the wavelet thresholding methods by averaging the estimation of all translations of the degraded signal the wavelet coefficients of the original and translated signals can be very different, and they are not related by a simple translation or permutation. Zhuang and Baras [10] studied the problem of choosing an image-based customized wavelet basis with compact support for image data compression and provided a general algorithm for computing the optimal wavelet basis.[11] Proposed an adaptive shrinkage denoising scheme by using neighbourhood characteristics. They claimed that their new scheme produced better results than Donoho's methods [3].

ISSN 2278 - 3091
International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE), Vol.2 , No.5, Pages : 01-08 (2013)
Special Issue of ICACET 2013 - Held during October 14-15, 2013, Kuala Lumpur, Malaysia

IMAGE FILTERING USING WAVELET TRANSFORMATION

Filtering operations in the wavelet domain can be subdivided into linear and nonlinear methods,the main principle in wavelet technique is using decomposition and reconstruction algorithm of signal, wavelet function processes the high-frequency signal according to a filtering scheme. The algorithm starts with a wavelet decomposition applied over the time series. The result, the coefficient array, is divided into two parts: an approximation coefficient vector and a detail coefficient vector, in a rough scale [12]. The following subsections show different types of wavelet filters.

Linear Filters

Linear filters such as Wiener filter in the wavelet domain yield optimal results when the signal corruption can be modeled as a Gaussian process and the accuracy criterion is the mean square error "MSE" [13],[14]. However, designing a filter based on this assumption frequently results in a filtered image that is more visually displeasing than the original noisy signal, even though the filtering operation successfully reduces the MSE. In [15] a wavelet-domain spatially adaptive FIR Wiener filtering for image denoising is proposed where Wiener filtering is performed only within each scale and intrascale filtering is not allowed.

Non-Linear Threshold Filtering

The most investigated domain in denoising using wavelet transform is the non-linear coefficient thresholding based methods. The procedure exploits sparsity property of the wavelet transform and the fact that the wavelet transforms maps white noise in the signal domain to white noise in the transform domain. Thus, while signal energy becomes more concentrated into fewer coefficients in the transform domain, noise energy does not. It is this important principle that enables the separation of signal from noise [16]. The procedure in which small coefficients are removed while others are left untouched is called hard thresholding. But the method generates spurious blips, better known as artifacts, in the images as a result of unsuccessful attempts of removing moderately large noise coefficients. In order to overcome the demerits of hard thresholding, wavelet transform using soft thresholding was introduced in[17]. In this scheme, coefficients above the threshold are shrunk by the absolute value of the threshold itself. Similar to soft thresholding, other techniques of applying thresholds are semi-soft thresholding and Garrote thresholding [18]. Most of thewavelet shrinkage literature is based on methods for choosing the optimal threshold which can be adaptive or non-adaptive to the image.

Non-Adaptive Thresholds

One of the most known algorithms in non–adaptive threshold is VisuShrink [17] which depends only on number of data points. It has asymptotic equivalence suggesting best performance in terms of "MSE" when the number of pixels reaches infinity. VISUShrink is known to yield overly smoothed images because its threshold choice can be unwarrantedly large due to its dependence on the number of pixels in the image. In the paper detailed explanations will be presented about VisuShrink.

Adaptive Thresholds

SUREShrink uses a hybrid of the universal threshold and the "SURE" (Stein's Unbiased Risk Estimator) threshold and performs better than VISUShrink. BayesShrink [18], [19] minimizes the Bayes' Risk Estimator function assuming Generalized Gaussian prior and thus yielding data adaptive threshold. BayesShrink outperforms SUREShrink most of the times. Cross validation [20] replaces wavelet coefficient with the weighted average of neighborhood coefficients to minimize generalized cross validation "GCV" function providing optimum threshold for every coefficient.

There are two primary thresholding methods: hard thresholding and soft thresholding [14]. Hardthresholding operator is defined as:

$$D(U,\lambda) = U \quad if\,|U| > \lambda \qquad (1)$$
$$D(U,\lambda) = 0 \quad otherwise$$

Soft thresholding operator is defined as:

$$D(U,\lambda) = (sgn(U)*max(0, |U| \geq \lambda)) \qquad (2)$$

The Method Noise

All denoising methods depend on a filtering parameter (*h*). This parameter measures the degree of filtering applied to the image. For most methods, the parameter (*h*) depends on an estimation of the noise variance σ^2. One can define the result of a denoising method D_h as a decomposition of any image *v* as:

$$v = Dhv + n(Dh, v) \qquad (3)$$

Where $D_h v$ is smoother than *v*, and *n(Dh, v)* is the noise guessed by the method.

It is not enough to smooth*v* to ensure that $n(D_h, v)$ will look like a noise. More methods are actually not content with a smoothing but try to recover lost information in *n(Dh, v)* [21], [22]. So the focus is on $n(D_h, v)$. Let *u* be a not necessarily noisy image and D_h a denoising operator depending on *h*. Then method noise can be defined as the image difference

$$n(Dh, u) = u - Dh(u) \qquad (4)$$

This method noise should be as similar to a white noise as possible. In addition, since the original image *u* should not to be altered by denoising methods, the method noise should be as small as possible for the functions with the right regularity. According to the preceding discussion, four criteria will be taken into account in the comparison of denoising methods:

- A display of typical artifacts in denoised images.
- A formal computation of the method noise on smooth images, evaluating how small it is in accordance with image local smoothness.
- A comparative display of the method noise of each method on real images with σ = 2.5. The noise standard deviation smaller than 3 are subliminal, and it is expected that most digitization methods allow themselves this kind of noise.
- A classical comparison receipt based on noise simulation: it consists of taking a good quality image, adding Gaussian white noise with known σ, and then computing the best image recovered from the noisy one by each method.

ISSN 2278 - 3091
International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE), Vol.2 , No.5, Pages : 01-08 (2013)
Special Issue of ICACET 2013 - Held during October 14-15, 2013, Kuala Lumpur, Malaysia

On top of this, in two cases, a proof of asymptotic recovery of the image can be obtained by statistical arguments.

Soft and Hard Thresholding

The threshold plays an important role in the denoising process. Fig. 2 demonstrates the hard and soft thresholding functions. Finding an optimum threshold is a tedious process. A small threshold value will retain the noisy coefficients whereas a large threshold value leads to the loss of coefficients that carry image signal details. Normally, hard thresholding and soft thresholding techniques are used for such de-noising process. Hard thresholding is a keep or kill rule whereas soft thresholding shrinks the coefficients above the threshold in absolute value. It is a shrink or kill rule. Famous algorithms will be presented and explained in details in the next section [23].

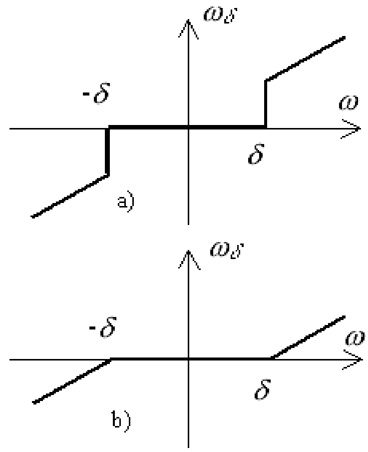


Fig.2. (a) Hard Thresholding, (b) Soft Thresholding Functions

WAVELET DENOISING ALGORITHMS

Bayes Shrink

The Bayes Shrink method is effective for images including Gaussian noise [24]. The observation model is expressed as follows:

$$Y = X + V \quad (5)$$

Here *Y* is the wavelet transform of the degraded image, *X* is the wavelet transform of the original image, and *V* denotes the wavelet transform of the noise components following the Gaussian distribution $N(0, \sigma v^2)$. Here, since *X* and *V* are mutually independent, the variances σy^2 , σx^2 and σv^2 of *y*, *x* and *v* are given by:

$$\sigma y^2 = \sigma x^2 + \sigma v^2 \quad (6)$$

It has been shown that the noise variance σy^2 can be estimated from the first decomposition level diagonal sub-band "HH1" by the robust and accurate median estimator [25].

$$\sigma^{\wedge}{}_v = \frac{media(|HH_1|)}{0.6745} \quad (7)$$

The variance of the sub-band of degraded image can be estimated as:

$$\sigma y^2 = \frac{1}{M}\sum_{m=1}^{M} A_m{}^2 \quad (8)$$

Where A_m are wavelet coefficients of sub-band under consideration, *m*is the total number of wavelet coefficient in that sub-band. The Bayes shrink thresholding technique performs soft thresholding, with adaptive data driven, sub-band and level dependent near optimal threshold given by [6]:

$$TBS = \begin{cases} \frac{\sigma v^2}{\sigma x^2} & if \quad \sigma_v{}^2 < \sigma_y{}^2 \\ max\{|A_m|\} & otherwise \end{cases} \quad (9)$$

Where $\sigma x = \sqrt{max(\sigma_y{}^2 - \sigma_v{}^2, 0)}$(10)

Normal Shrink

The optimum threshold value for the Normal Shrink "TN"is given by [26]:

$$TN = \frac{\lambda \sigma_v{}^2}{\sigma_v} \quad (11)$$

Where, the parameter λ is given by the following equation:

$$\lambda = \sqrt{log\left(\frac{L_k}{J}\right)} \quad (12)$$

$L_k$is the length of the sub-band at k[th] scale. And, *J* is the total number of decomposition. *σv* is the estimated noise variance, calculated by equation (7) and *σy* is the standard deviation of the sub-band of noisy image, calculated by using equation (8).Normal Shrink also performs soft thresholding with the data driven sub-band dependent threshold "TN", which is calculated by equation (11).

Neigh Shrink

Let $g = \{g_{ij}\}$ will denote the matrix representation of the noisy signal [23] Then, *w(Wg)* denotes the matrix of wavelet coefficients of the signal under consideration. For every value of w_{ij}, letB_{ij} is a neighboring window around w_{ij}, w_{ij} denotes the wavelet coefficient to be shrinked. The neighboring window size can be represented as *LxL*, where *L* is a positive odd number. A3x3 neighbouring window centered at the wavelet coefficient to be shrinked is shown in Fig. 3.

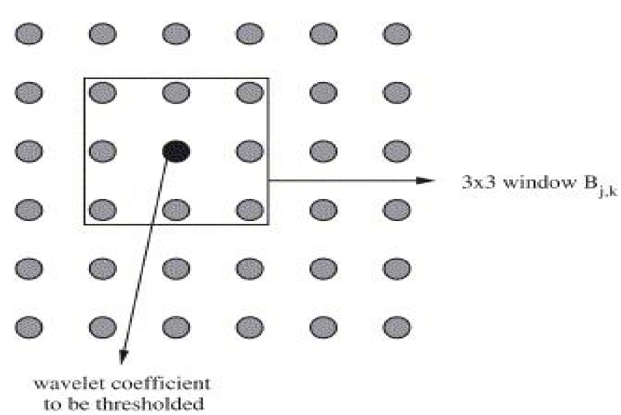


Fig. 3. An Illustration of theNeighbouring Window of Size 3 3 Centered at The Wavelet Coefficient to be Shrinked [28]

ISSN 2278 - 3091
International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE), Vol.2 , No.5, Pages : 01-08 (2013)
Special Issue of ICACET 2013 - Held during October 14-15, 2013, Kuala Lumpur, Malaysia

Let
$s_{ij}=\sum_{(k,l)\epsilon B_{ij}} w_{kl}$(13)

The corresponding terms will be omitted in the summation when the above summation has pixel indexes out of the wavelet sub-band range. The shrinked wavelet coefficient according to the Neigh shrink is given by this formula [29].

$w'_{ij} = w_{ij}\,\beta_{ij}$(14)

The shrinkage factor β_{ij} can be defined as:

$\beta_{ij} = \left(1 - \frac{T_{UNI}{}^2}{s_{ij}{}^2}\right)_+$ (15)

Here, the + sign at the end of the formula means to keep the positive value while set it to zero when it is negative and is the universal threshold, which is defined as [4]:

$T_{UNI} = \sqrt{2\sigma^2 ln\,(n)}$(16)

Different wavelet coefficient sub-bands are shrinked independently, but the universal threshold T_{UNI} and neighboring window size L kept unchanged in all sub-bands. The estimated denoised signal $\mathrm{f}' = \mathrm{f}'_{\mathrm{ij}}$ is calculated by taking the inverse wavelet transform of the shrinked wavelet coefficients $w`_{ij}$ i.e$f' = W^{-1}\,(w')$.

SureShrink

A threshold chooser based on Stein's Unbiased Risk Estimator "SURE" was proposed by Donoho and Johnstone [15] and is called as SureShrink. It is a combination of the universal threshold and the "SURE" threshold. This method specifies a threshold value t_j for each resolution level j in the wavelet transform which is referred to as level dependent thresholding [27]. The goal of SureShrink is to minimize the mean squared error, defined as [28].

$MSE=\frac{1}{n^2}\sum_{x,y=1}^{n}(z(x,y) - s(x,y))^2$(17)

Where $z(x,y)$ is the estimate of the signal while $s(x,y)$ is the original signal without noise and n is the size of the signal. SureShrink suppresses noise by thresholding the empirical wavelet coefficients. The SureShrink threshold t^* is defined as:

$t^*=min\left(t,\sigma\sqrt{2logn}\right)$(18)

Where t denotes the value that minimizes Stein's Unbiased Risk Estimator, σ is the noise variance computed from Equation (7), and n is the size of the image. SureShrink follows the soft thresholding rule. The thresholding employed here is adaptive, i.e., a threshold level is assigned to each dyadic resolution level by the principle of minimizing the Stein's Unbiased Risk Estimator for threshold estimates. It is smoothness adaptive which means that if the unknown function contains abrupt changes or boundaries in the image, the reconstructed image also does.

VisuShrink

Visushrink is thresholding by applying the Universal threshold proposed by Donoho and Johnstone [13]. This threshold is given by:

$\sigma\sqrt{2\ log\ M}$ (19)

where σ is the noise variance and M is the number of pixels in the image.It is proved in [3] that the maximum of any M valuesindependent and identically distributed "*i.i.d*" as $N(0, \sigma 2)$ will be smaller than the universal threshold with high probability, with the probability approaching 1 as M increases. Thus, with high probability, a pure noise signal is estimated as being identically zero. However,for denoising images, Visushrink is found to yield an overly smoothed estimate. This is because the universal threshold "UT" is derived under the constraint that with high probability the estimate should be at least as smooth as thesignal. So the "UT" tends to be high for large values of M, killing many signal coefficients along with the noise. Thus, the threshold does not adapt well to discontinuities in the signal.

Bivariate Shrink

New shrinkage function which depends on both coefficient and its parent yield improved results for wavelet based image denoising [29]. Here, then modify the Bayesian estimation problem as to take into account the statistical dependency between a coefficient and its parent. Let $w2$ represent the parent of $w1$ ($w2$ is the wavelet coefficient at the same position as w1, but at the next coarser scale.) Then

$y1=w1+n1$ (20)

$y2=w2+n2$ (21)

Where y1 and y2 are noisy observations of w1 and w2 and n1 and n2 are noise samples, it can be written as:

$y=w + n$ (22)

$y= (y1, y2)$ (23)

$w= (w1, w2)$ (24)

$n= (n1, n2)$ (25)

According to bays rule allows estimation of coefficient can be found by probability densities of noise and prior density of wavelet coefficient. Assume that noise is Gaussian then it can be written as:
$p_n(n)= ½*\pi*(\sigma_n^2)*exp(-n_1^2+n_2^2/2\sigma_n^2)$ (26)

This equation is equivalent to solving following equations

$y_1 - \frac{w`_1}{\sigma_n{}^2} + f_1(w`) = 0$ (27)

$y_2 - \frac{w`_2}{\sigma_n{}^2} + f_2(w`) = 0$(28)

Here $f1$and $f2$ represent the derivative of $f(w)$ with respect to $w1$and $w2$ respectively. It is clear to know $f(w)$ can be written as:
$f(w)=log(pw(w))$ (29)

$w`_1 = \frac{\left(\left(\sqrt{y^2{}_1+y^2{}_2}-\sqrt{3}\frac{\sigma^2{}_n}{\sigma}\right)+y_1\right)}{\sqrt{y^2{}_1+y^2{}_1}}$(30)

ISSN 2278 - 3091

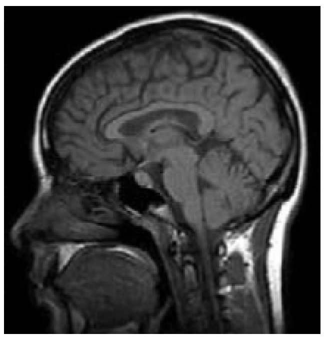
International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE), Vol.2 , No.5, Pages : 01-08 (2013)
Special Issue of ICACET 2013 - Held during October 14-15, 2013, Kuala Lumpur, Malaysia

EXPEREMINTAL RESULTS

Experiments have been conducted using Matlab. The testing images are Lena andMRIScan of size 256 × 256 at different noise levels σ=10, 20, 30 and 35. Fig. 4 and Fig. 5 show the free noisy images and the relative images with different denoising techniques. Softthresholding has been used over hard thresholding because it gives more visually pleasant images as compared to hard thresholding; reason being the latter is discontinuous and yields abrupt artifacts in the recovered images especially when the noise energy is significant. For comparison, VisuShrink, NeighShrink, NormalShrink, Bivariate shrink, SureShrink are implemented to denoise the noisy images. Using soft-thresholdingdenoising technique [30]. The wavelet filter length in these experiments is set to 8. The wavelet transform employs Daubechies' least asymmetric compactly supported wavelet with eight vanishing moments. Table1and Table2present the PSNR and resulting images of different algorithms as mentioned earlier forLena andMARIScan at different noise levels.

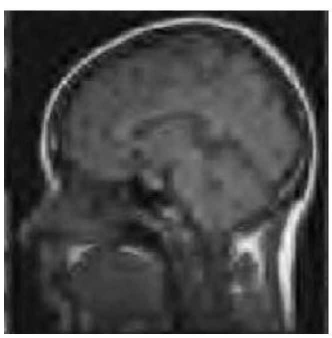
$$PSNR = -10\, log_{10} \frac{\sum_{ij}(B(i,j))-A(i,j))^2}{n^2 256^2} \quad (31)$$

Where $A\,(i, j)$ be the noise-free image and $B\,(i, j)$ the image corrupted with white noise, 256 is the image size.

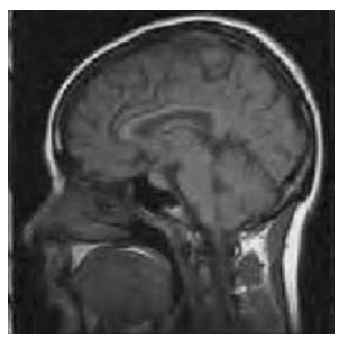


Noise-free image

Noisy image



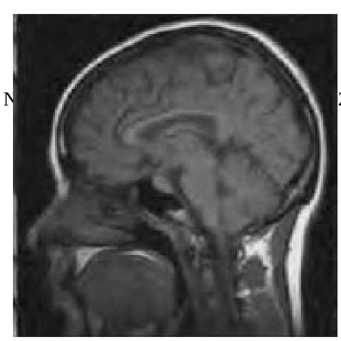
Bayes Shrink



NeighShrink

Fig.4. Im ... ds on a N ... 22 dB.

Bivariate Shrink



VisuShrink

ISSN 2278 - 3091

International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE), Vol.2 , No.5, Pages : 01-08 (2013)
Special Issue of ICACET 2013 - Held during October 14-15, 2013, Kuala Lumpur, Malaysia

Noise-free image

Noisy image

Bayes Shrink

NeighShrink

Bivariate Shrink

VisuShrink

Fig.4. Image denoising by Using Different Methods on a NoisyImage (Lena) with PSNR = 22 dB.

Table 2. The PSNR (dB) of the noisy images of MRIScan and the denoised images withdifferent denoising methods

Noisy image	VisuShrink	NeighShrink	Bayes Shrink	NormalShrink	SureShrink	Bivariate Shrink
28.14	26.85	32.41	33.05	30.74	33.46	**33.51**
22.12	24.08	31.07	28.97	25.05	29.34	**31.38**
18.60	22.74	**29.41**	26.77	21.60	27.20	28.60
16.10	21.91	**30.55**	25.33	19.11	25.92	27.94
14.14	21.34	**28.22**	24.26	17.17	24.79	26.51
12.58	20.90	24.01	23.40	15.58	23.68	**25.09**
11.24	20.50	22.12	22.68	14.23	22.90	**23.89**

ISSN 2278 - 3091

International Journal of Advanced Trends in Computer Science and Engineering (IJATCSE), Vol.2 , No.5, Pages : 01-08 (2013)
Special Issue of ICACET 2013 - Held during October 14-15, 2013, Kuala Lumpur, Malaysia

Table 1. The PSNR (dB) of the noisy images of Lena and the denoised images with different denoising methods

Noisy image	VisuShrink	NeighShrink	Bayes Shrink	NormalShrink	SureShrink	Bivariate Shrink
28.14	26.35	**33.80**	32.34	33.53	33.47	33.61
22.12	23.88	28.90	28.26	30.35	30.07	**30.38**
18.60	22.87	26.62	26.06	28.53	28.39	**28.60**
16.10	22.31	25.08	24.64	27.89	27.63	**27.94**
14.14	21.90	24.14	23.68	26.37	26.29	**26.50**
12.58	21.53	23.30	22.94	25.32	25.09	**25.40**
11.24	21.23	22.74	22.36	23.22	**24.42**	24.22

Potential for Future Research

After seeing all the above discussions and explanations, we can summarize some points about the type of thresholding (hard, soft). Hard wavelet threshold method noise is concentrated on the edges and high frequency features where the wavelet coefficient processed by the threshold value have discontinuous point on the threshold λ and - λ, which may cause Gibbs shock to the useful reconstructed signal. These structures lead to coefficients of large enough value but lower than the threshold, they are removed by the algorithm. On the other hand, the soft wavelet threshold method noise presents much more structure than the hard thresholding, but when the wavelet coefficients are greater than the threshold value, there will be a constant bias between the wavelet coefficients that have been processed and the original wavelet coefficients, making it impossible to maintain the original features of the images effectively. The literature search revealed that a lot of research has gone into using deferent imaging techniques using wavelet approaches via hard and soft thresholding. A lack of integration of these two techniques as mentioned above, however, is evident from the fact that the noise difficult to be eliminated completely from the noisy image, but on the other side of this fact we can minimize it as much as we can. The promising results of the new proposed method that depends on the semi soft threshold can reduce the noise and improve the quality and the quantity of the image, furthermore, in this algorithm a compromise has to be found between noise reduction and preserving significant signal details. In order to achieve a good performance in this respect, a denoisingalgorithm has to adapt to signal discontinuities especially in non-repeated, contours, texture and flat object structures.

CONCLUSION

Usually there will be various noises in the process of image acquisition. Image denoisingis paid more and more attention of scholars based on the theory of wavelet because wavelet transform has good local time-frequency, multi-scale and multi-resolution characteristics. This paper summarized and reviewed some algorithms and techniques that used to improve the image and found compromise between noise reduction and preserving significant signal details. Experimental results showed that in most cases Bivariate Shrink method gives better results than VisuShrink, NormalShrink, It should be mentioned that NeighShrink method presented outperformance in some cases especially with average of noise image "PSNR" in range between 14-17 dB.

REFERENCES

[1] A. Buades, B. Coll, and M. Jean-Michel. "Image and movie denoising by nonlocal means," *Journ`eeannuelle*, pp. 1-38, July 2006.

[2] S.Sulochana, R.Vidhya. "Image Denoising using Adaptive Thresholding in FrameletTransform Domain," *international Journal of Advanced Computer Science and Applications*,vol. 3, No. 9 April 2012.

[3] D. Donoho and I..Johnstone,"Ideal spatial adaption via waveletshrinkage",*Biometrika*,vol.81, pp 425-455, September 1994.

[4] S.Kother, S. Arumuga, M.Mohamed."Image De-noising using Discrete Wavelet transform,"*International Journal of Computer Science and Network Security*, vol.8 No.1, January 2008.

[5] T. Bui, G. Chen, "Translation invariant denoising using multiwavelets," *IEEE Trans Signal Process.*Vol. 12, pp. 3414–3420, 1998.

[6] R.R. Coifman, D.L. Donoho, "Translation invariant de-noising,in: A. Antoniadis Wavelets andStatistics,"*Springer LectureNotes in Statistics*, vol. 103, Springer, New York, pp. 125–150, 1995.

[7] E.P. Simoncelli, E.H. Adelson, "Noise removal via Bayesian wavelet coring,"*The Third International Conference on Image Processing*, Lausanne, Switzerland, 16-19 September 1996.

[8] Y. Mallet, D. Coomans, J. Kautsky, O. DeVel, "Classification using adaptive wavelets for feature extraction,"*IEEE Trans. Pattern Anal. Mach. Intell.*Vol. 19 (10), pp. 1058–1066, 1997.

[9] M.K. Mihcak, I. Kozintsev, K. Ramchandran, P. Moulin, "Lowcomplexity image denoising based on statistical modeling of wavelet coefficients," *IEEE Signal Process*. Lett. 6 ,vol. 12 ,pp. 300–303 1999.

[10] Y. Zhuang, J.S. Baras, "Constructing optimal wavelet basis for image compression," *IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 4, pp. 2351–2354, 1996.

[11] M.S. Crouse, R.D. Nowak, R.G. Baraniuk, "Wavelet-based signal processing using hidden Markov models,"*IEEE Trans.Signal Process.* 46 vol.4 ,pp. 886–902, 1998.

[12] L.Ebadi, H. Z. M. Shafri, S. B. Mansor, R.Ashurov," A review of applying second-generation wavelets for noise removal from remote sensing data,"*Environ Earth Sci* .DOI 10.1007/s12665-013-2325-z. Springer-Verlag Berlin Heidelberg. 2013.

[13] V. Strela. "Denoising via block Wiener filtering in wavelet domain," *In 3rd European Congress of Mathematics*, Barcelona,BirkhäuserVerlag, July 2000.

[14] H. Choi and R. G. Baraniuk, "Analysis of wavelet domain Wiener filters,"*in IEEE Int.Symp. Time-Frequency and Time-Scale Analysis*,Pittsburgh, October 1998.

[15] D. L. Donoho, "De-noising by soft-thresholding,"*IEEE Trans. InformationTheory*, vol.41,pp.613-627,May1995.

[16] S. Jangra, S. Kumar,"A New Threshold Function for Image Denoising based on Wavelet Transform" *International Journal of Engineering and Mathematical Science.* Vol. 1, Issue – 1, pp.60-65, June 2012.

[17] I. K. Fodor, C.Kamath, "Denoising through wavlet shrinkage: An empirical study," *Center for applied science computing Lawrence Livermore National Laboratory*, July 2001.

[18] E. P. Simoncelli and E. H. Adelson. Noise removal via Bayesian wavelet coring. In Third Int'l Conf on Image Proc, volume I, pages 379-382, Lausanne. IEEE Signal ProcSociety.Sep(1996)

[19] H. A. Chipman, E. D. Kolaczyk, and R. E. McCulloch,"Adaptive Bayesian wavelet shrinkage,"vol. 92, No 440, pp.1413-1421 December 1997.

[20] M. Jansen, "Wavelet thresholding and noise reduction" Ph.D. dissertation,Dept. Elect. Eng., KatholiekeUnivLeuven,April 2000.

[21] F. Malgouyres, "A noise selection approach of image restoration,"*in Proceedings of Wavelet Applications in Signal and Image Processing IX, SPIE Proc*. Ser. 4478, SPIE, Bellingham, WA, pp. 34–41,2001.

[22] S. Osher, M. Burger, D. Goldfarb, J. Xu, and W. Yin, "An iterative regularization method for total variation-based image restoration,"*Multiscale Model. Simul, 4*, pp. 460–489, 2005.

[23] R.Sihag, R. Sharma, V.Setia, "Wavelet Thresholding for Image De-noising," *International Conference on VLSI, Communication & Instrumentation* Proceedings published by International Journal of Computer Applications, pp. 20-24, 2011.

[24] N.Dewangan, A. D.Goswami, " Image denoising using wavletthresholding methods,"*Int. J. of Engg. Sci. & Mgmt. (IJESM)*, Vol. 2, Issue 2:, 271 -275, April-June 2012.

[25] S. Grace Chang, Bin Yu and M. Vattereli, "Adaptive Wavelet Thresholding for Image Denoising and Compression," *IEEE Trans.Image Processing*, vol. 9, pp. 1532-1546, Sept.2000.

[26] L. Kaur, S. Gupta and R.C.Chauhan, "Image denoising using wavelet thresholding,"*Proceeding of the Third Indian Conference On Computer Vision, Graphics & Image*.2002.

[27] A. Antoniadis, J. Bigot, "Wavelet Estimators in Nonparametric Regression: A Comparative Simulation Study," *Journal of Statistical Software,* vol. 6, I. 06, 2001.

[28] S. G. Chang, B. Yu and M.Vetterli, "Adaptive Wavelet Thresholding for Image Denoising and Compression," *IEEE Trans. Image Processing,*Vol 9, No. 9, pp. 1532-1546.Sept (2008)

[29] G. Y. Chen and T. D. Bui, "Multi-wavelet De-noising using Neighboring Coefficients," IEEE Signal Processing Letters, vol.10, no.7, pp.211-214,2003.

[30] T.Hui, L.Zengli, C. Lin and C.Zaiyu,"Wavelet Image Denoising Based on The New Threshold Function," *Proceedings of the 2nd International Conference on Computer Science and Electronics Engineering*. Published by Atlantis Press, Paris, France. pp. 2749-2752, 2013.