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Investigation on Adaptive Gain in MRAC for First Order Systems

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ABSTRACT

MRAC is a popular adaptive scheme which guarantees the aspect of high dynamic tracking and stability. Incorporating these mutually contradictory features in a control system has been a sustained challenging aspect in the system design. The ability of high tracking in the transient response and accurate performance is the steady state relies on the crucial selection of adaptive parameter in MRAC. Adaptive updation law is derived when both time constant and DC gain for the first order system is subjected to variations. Methodology has been investigated to select the appropriate value of adaptive gain. Methodology is based on rigorous mathematical background and simulated results. The improved performance is demonstrated on first order system under parameters variations.

Key Words: MRAC, MIT Rule, Adaptive Gain.

INTRODUCTION

Design of Adaptive controllers has given a significant contribution in the era of modern control design. Conventional controllers design are model based designed. Adaptive Control covers a set of techniques which provide a systematic approach for automatic adjustment of controllers in real time, in order to achieve or to maintain a desired level of control system performance when the parameters of the plant dynamic model are unknown and/or change in time. For example, as an aircraft flies, its mass will slowly decrease as a result of fuel consumption; a control law is needed that adapts itself to such changing conditions. The distinguishing aspect of Adaptive control is that it does not need a priori information about the bounds on these uncertain or time-varying parameters [2]. The design is proven to be close to real time conditions:

An adaptive controller is a controller with adjustable parameters and a mechanism for adjusting the parameters. An adaptive control system can be thought of as having two loops as in Figure 1. One loop is a normal feedback loop with the process (plant) and controller. The other loop is a parameter adjustment loop [1].

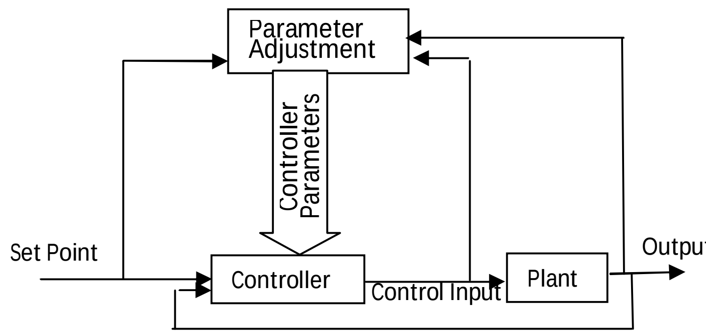

Fig. 1 Block Diagram of Adaptive Controller

MODEL REFERENCE ADAPTIVE CONTROL

Model reference adaptive control is a powerful adaptive control scheme, which has a rigorous and systematic theoretical foundation, an attractive and promising application perspective and an easy and concise design procedure. In MRC, a good understanding of the plant and the performance requirements allow the designer to come up with a model, referred to as the reference model, that describes the desired I/O properties of the closed-loop plant[Fig.2]. The performance requirements are specified in terms of the reference model, which describes the desired I/O properties of the closed-loop plant. The reference model is designed so that for a given reference input signal the output of the reference model represents the desired response the plant output should follow. The feedback controller is designed so that all signals are bounded and the closed-loop plant transfer function is equal to reference model. This transfer function matching guarantees that for any given reference input, the tracking error which represents the deviation of the plant output from the desired trajectory converges to zero with time.

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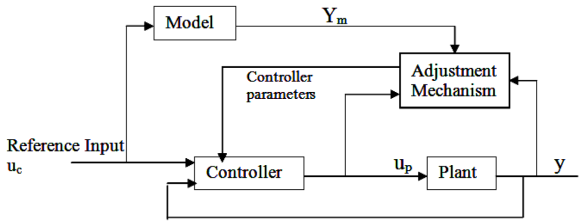

Fig. 2. Block diagram for MRAC

Model reference adaptive control can be designed using either state feedback or output feedback; output feedback model reference control is more challenging and has more potential for applications. When the plant parameters are unknown, adaptive laws are needed to update the parameters of a model reference controller. The main issues in model reference adaptive control include; 1) Controller parameterization 2) Error model derivation 3) A priori plant knowledge specifications 4) Adaptive law design 5) Stability, tracking and robustness [3,4].

GRADIENT METHOD (MIT RULE)

A popular method for computing the approximate sensitivity functions is the so-called **MIT** rule. MIT rule changes the parameters based upon the gradient of the error with respect to that parameter. The parameters are changed in the direction of the negative gradient of the error. This means that if the error, with respect to a specific parameter, is increasing then by the MIT rule the value of that parameter will decrease by the equation given below:

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta}$$

In this equation: $\frac{d\theta}{dt}$ is the incremental change to make to parameter θ. γ is the adaptation rate. e is the error between the outputs of the plant and the model. $\frac{\partial e}{\partial \theta}$ is the rate of change of the error with respect to the parameter θ.

MRAC schemes can be characterized as direct or indirect and with normalized or unnormalized adaptive laws. In direct MRAC, the parameter vector of the controller is updated directly by an adaptive law, whereas in indirect MRAC, controller parameters is calculated at each time by solving a certain algebraic equation that relates controller parameters with the on-line estimates of the plant parameters.

An adaptive controller may be considered as a combination of an on-line parameter estimator with a control law that is derived from the known parameter case. The way this combination occurs and the type of estimator and control law used gives rise to a literature of adaptive control the on-line parameter estimator has often been referred to as the adaptive law, update law or adjustment mechanism. The design of the adaptive law is crucial for the stability properties of the adaptive controller.

From the simple MIT rule the Updation Mechanism is as follows:

In this rule, a cost function is defined as,

$$J(\theta) = e^2/2 \qquad \ldots\ldots\ldots\ldots \quad (1)$$

Where e is the error between the outputs of plant and the model, and θ is the adjustable parameter. Parameter θ is adjusted in such a fashion so that the cost function can be minimized to zero. For this reason, the change in the parameter θ is kept in the direction of the negative gradient of J, that is

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} \qquad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

From Eq. (1) &(2),

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta} \qquad \ldots\ldots\ldots\ldots \quad (3)$$

Where, the partial derivative term is called as the sensitivity derivative of the system. This term indicates how the error is changing with respect to the parameter θ. And eq. (2) describes the change in the parameter θ with respect to time so that the cost function J(θ) can be reduced to zero. Here γ is a positive quantity which indicates the adaptation gain of the controller[5].

The plant under consideration is a first order system. A first order system is characterized by its time constant and gain. The general representation for the transfer function is given by : $\frac{K_p}{1+sT_p}$.

T_p is the time constant and K_p is the DC gain of the system. These parameters are subjected to variations to validate the performance of proposed control scheme. Our goal is to design a controller so that the plant could track any first order reference model[4].

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Defining the error signal between the plant and reference model. $e(t) = y_p(t) - y_m(t)$.. (4)
$E(s) = G_p(s)U_p(s) - G_m(s)U_c(s)$ ………..….. (5)
Defining a control law:
$u_p(t) = \theta(t) * u_c(t)$ …………………..…. (6)
$\frac{\partial e}{\partial \theta} = G_p(s)U_c(s)$ …………………….. (7)
For a first order system the parameters of the plant are expressed in terms of the model:
$G_p(s) = G_m(s\times\frac{T_p}{T_m}) \times\frac{K_p}{K_m}$ ………….. (8)
From 3 and 8, and scaling property of laplace transforms[6].
$L\{f(t)\} \leftrightarrow F(s)$
$F(ks) \leftrightarrow \frac{1}{k}L\{f(\frac{t}{k})\}$

$$\frac{d\theta}{dt} = -\gamma e \frac{T_m}{T_p} y_m(\frac{T_m}{T_p}t) \times \frac{K_p}{K_m}$$

$$= -\gamma \frac{T_m}{T_p}\frac{K_p}{K_m} e = \gamma_v e y_m(t) \quad \text{…. (9)}$$

$$\gamma_v = -\gamma \frac{T_m}{T_p}\frac{K_p}{K_m}\frac{y_m\left(\frac{T_m}{T_p}t\right)}{y_m(t)} \quad \text{…….. (10)}$$

Equation 10 gives the condition for absolute tracking. According to the MRAC scheme the value of adaptation gain is a constant value. Appropriate selection of this gain is very crucial in the tracking performance.
We analyze the equation to arrive at appropriate selection of adaptation gain.

ANALYSIS OF ADAPTIVE GAIN EQUATION

Adaptive Gain, γ_v is a factor dependent on system and plant parameters. For a first order system, the factor given by $\gamma \frac{T_m}{T_p}\frac{K_p}{K_m}$ is a constant factor, which can provide the basis for proper selection, the other factor is variable, the effect of this factor can be appropriately nullified by the proper estimation of the adaptive gain. The control scheme is represented in Figure 3.

ILLUSTRATION WITH CASE STUDIES

The plant parameters are considered as unity. Therefore, the time constant of the plant T_p = 1 and DC gain K_p = 1.
The reference model parameters are taken in four cases with increased and decreased values of plant parameters.

Case1: DC gain K_m = 10 and time constant T_m=1
Case2: DC gain K_m = 0.1 and time constant T_m = 1
Case3: DC gain K_m = 1 and time constant T_m = 0.1
Case4: DC gain K_m = 1 and time constant T_m = 10

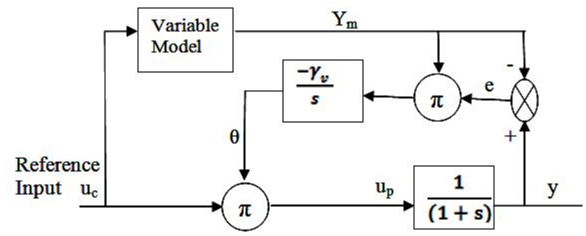
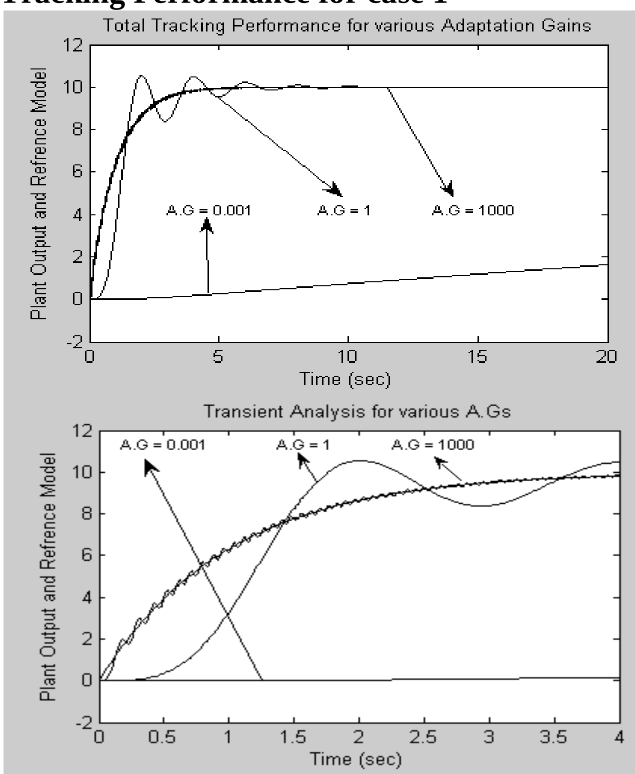


Fig3. Block Diagram of Adaptive Scheme for First Order System

EFFECT OF ADAPTATION GAIN, γ_v ON THE SYSTEM PERFORMANCE

Case 1: DC gain K_m = 10 and time constant T_m = 1. The constant factor in the Adaptive equation $\gamma \frac{T_m}{T_p}\frac{K_p}{K_m}$ is calculated for this case. The selection of γ as 10 is maintained for all cases. Therefore the value for γ_v as obtained from the expression $\gamma_v = \gamma \frac{T_m}{T_p}\frac{K_p}{K_m}$ is 1. A factor of 1000 is applied to see the effect in the tracking performance, therefore system is simulated for gamma = 0.001, 1 and 1000.

Tracking Performance for case 1



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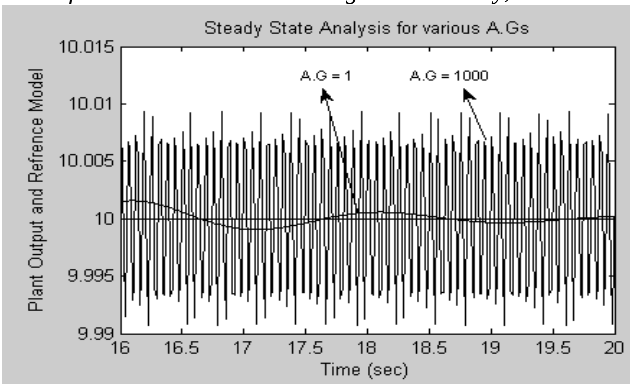


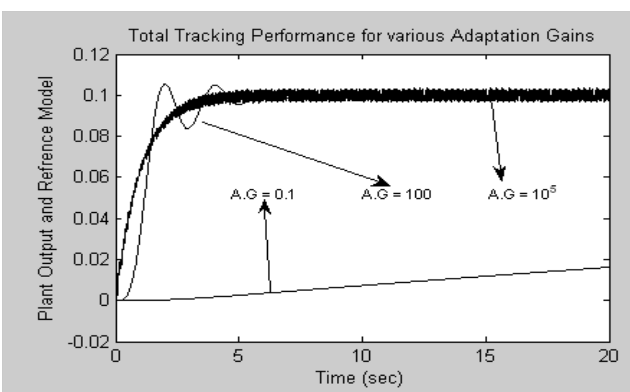
Fig4. Tracking Performance for case 1

Simulation is carried upto 20 Seconds(Fig. 4), with step input, the transient performance is examined from 0-4seconds, for Adaptive Gain (AG) =1000, the tracking is fast but there are sustained oscillations in the band of 1% of the steady state value. On the other hand, for Adaptive Gain (AG) = 0.001, there is no tracking or very very slow tracking. The calculated value of γ_v as 1 is giving peak overshoot less than 1% and a very stable performance in the steady state with a tolerance less that 0.3%.

Tracking Performance for case 2

For the settings of case 2, the value for γ_v as obtained from the expression $\gamma_v = \gamma \frac{T_m}{T_p}\frac{K_p}{K_m}$ is 100

A factor of 1000 is applied to see the effect in the tracking performance, therefore system is simulated for gamma = 0.1, 100 and 10^5.



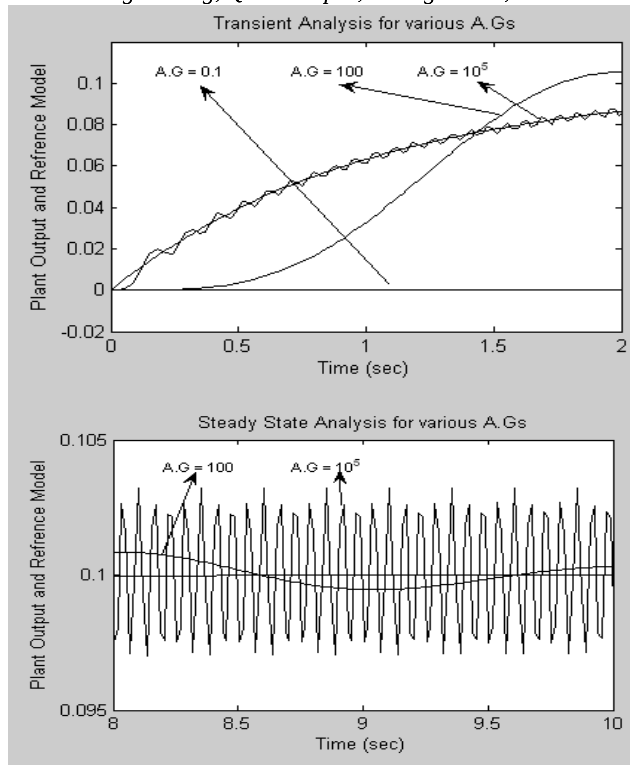


Fig5. Tracking Performance for case 2

Simulation is carried upto 20 Seconds (Fig 5), with step input, the transient performance is examined from 0-2seconds, for Adaptive Gain (AG) = 10^5, the tracking is fast but there are sustained oscillations in the band of .5% of the steady state value. On the other hand, for Adaptive Gain (AG) = 0.1, there is no tracking observed. The calculated value of γ_v as 1 is giving peak overshoot less than 1% and a very stable performance in the steady state with a tolerance less that 0.3%.

Tracking Performance for case 3

For the settings of case 3, the value for γ_v as obtained from the expression $\gamma_v = \gamma \frac{T_m}{T_p}\frac{K_p}{K_m}$ is 1

A factor of 1000 is applied to see the effect in the tracking performance, therefore system is simulated for gamma = 0.001, 1 and 1000.

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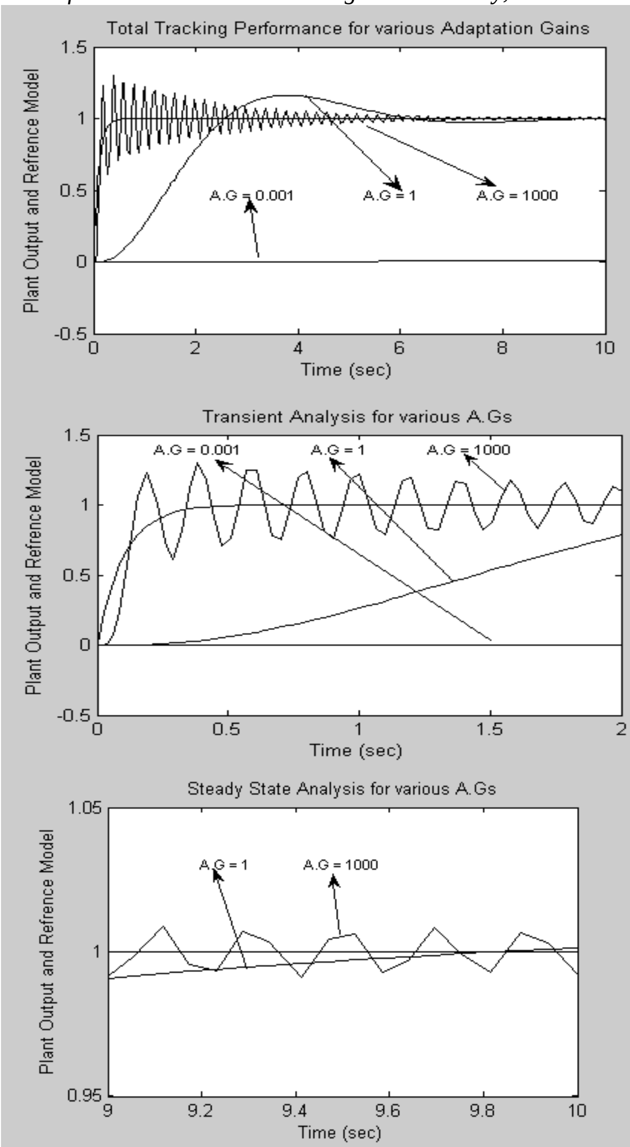

Fig6. Tracking Performance for case 3

Simulation is carried upto 10 Secs (Fig. 6), with step input, the transient performance is examined upto 2 seconds, for AG = 1000, the tracking is fast but there are sustained oscillations in the band of 3% of the steady state value. On the other hand, for AG = 0.001, there is no visible tracking.

The calculated value of γ_v as 1 is giving peak overshoot less than 20% and progressively decreasing steady state error with a tolerance less that 2%.

Tracking Performance for case 4

For the settings of case 4, the value for γ_v as obtained from the expression $\gamma_v = \gamma \frac{T_m}{T_p}\frac{K_p}{K_m}$ is 100. A factor of 1000 is applied to see the effect in the tracking performance, therefore system is simulated for gamma = 0.1, 100 and 10^5.

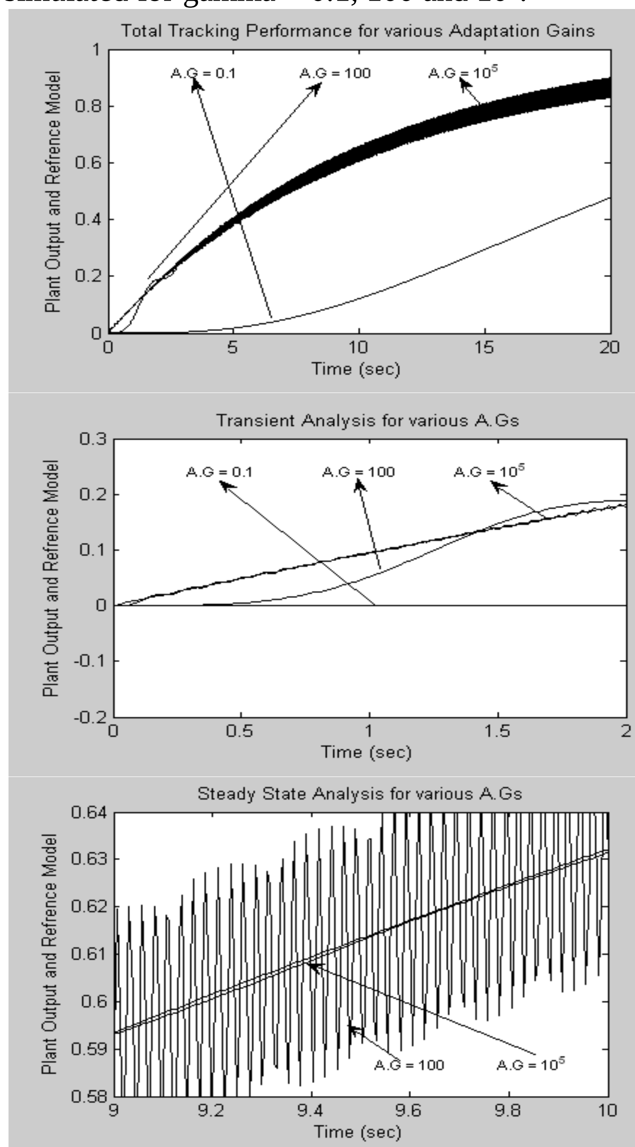

Fig7. Tracking Performance for case 4

Simulation is carried upto 20 Secs, with step input, the transient performance is examined upto 2 seconds(Fig 7). For AG = 10000, the tracking is fast but there are sustained oscillations in the steady state value. On the other hand, for AG = 0.1, the tracking is very slow. The estimated value of γ_v as 100 is having good transient and steady state performance.

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OBSERVATION FROM THE CASES:

From all the cases it is evident that the output tracking performance significantly depends on the selection of gamma. The performance is observed for steady state and transient response. The corresponding regions of observation are magnified for accurate analysis.

The estimated value of Adaptation Gain in all the cases is giving good transient and steady state performance. With the higher values of gamma, the system is tracking very quickly, it is very anticipative, but the system is tending to be oscillative. Particularly the tendency to exhibit unstable steady state behavior is prominent, the oscillations are sustained.

For lower values the adaptation is slow, it is having poor transient performance, but it is tracking with the progressive reduction in the steady error, thereby resulting into a stable system.

DISCUSSIONS AND CONCLUSIONS

Here the methodology for selection on adaptation Gains is investigated. The involved aspects of stability and adaptability in MRAC for first order system are studied. This analysis is universally applicable for any plant verses model tracking. The crucial issue of selection of adaption gain is resolved with complete mathematical analysis and simulation study. The performance with respect to the adaptation gain is tested for different cases and scaled values of Adaptation Gain. The simulation results are analyzed through comparison graphs and the selection on the adaptation Gain is validated.

The analysis gives a scope for introducing a variable parameter for Adaptation Gain for first order systems.

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