Estimation of Adaptive Minimum Mean Square Equalizer for Blind Channel Equalization using Normalized LMS Algorithm

T. Saikumar[#], S.Venkatesh[#], G.Nagarjun[*], Anil Santi[*]
[#]Associate Professor, Dept of ECE , CMR Technical Campus, Hyderabad, T.S, India.
[*] Assistant Professor, Dept of ECE, CMRCollege of Engg & Technology, Hyderabad, T.S, India.
ara.sai437@gmail.com, venkateshswarna@gmail.com,nagsunny447@gmail.com, anil.santi@gmail.com

Abstract— **The adaptive algorithm has been widely used in the digital signal processing like channel estimation, channel equalization, echo cancellation, and so on. One of the most important adaptive algorithms is the NLMS algorithm. We present in this paper an multiple objective optimization approach to fast blind channel equalization. By investigating first the performance (mean-square error) of the standard fractionally spaced CMA (constant modulus algorithm) equalizer in the presence of noise, we show that CMA local minima exist near the minimum mean-square error (MMSE) equalizers. Consequently, Fractional Spaced CMA may converge to a local minimum corresponding to a poorly designed MMSE receiver with considerablely large mean-square error. The step size in the NLMS algorithm decides both the convergence speed and the residual error level, the highest speed of convergence and residual error level.**

Keywords: Fractional Spaced CMA, NLMS, Adaptive MMSE.

I.   INTRODUCTION

 Blind equalization has the potential to improve the efficiency of communication systems by eliminating training signals. Difficulties of its application in wireless communications, however, are due largely to the characteristics of the propagation media - multipath delays and fast fading. The challenge is achieving blind equalization using only a limited amount of data.

A widely tested algorithm is the constant modulus algorithm (CMA). In the absence of noise, under the condition of the channel invertibility, the CMA converges globally for symbol-rate IIR equalizers and fractionally spaced FIR equalizers. It is shown in [9] that CMA is less affected by the ill-conditioning of the channel. However, Ding *et. al.* [2] showed that CMA may converge to some local minimum for the symbol rate FIR equalizer. In the presence of noise, the analysis of convergence of CMA is difficult and little conclusive results are available. Another drawback of CMA **is** that its convergence rate may not be sufficient for fast fading channels. Another approach to the blind equalization is based on the blind channel estimation. Some of the recent eigen structure-based channel estimations require a relatively smaller data size comparing with higher-order statistical methods. However the asymptotic performance of these eigen structure-based schemes is limited by the condition of the channel [12, 13]. Specifically, the asymptotic normalized mean square error (ANMSE) is lower bounded by the condition number of the channel matrix. Unfortunately, frequency selective fading channels

with long multipath delays often result in ill-conditioned channel matrices.

II.   BLIND CHANNEL EQUALIZATION AND TYPES

The field of blind channel equalization has been existence for a little over twenty years. Research during this time has centered on developing new algorithms and formulating a theoretical justification for these algorithms. Blind channel equalization is also known as a self-recovering equalization. The objective of blind equalization is to recover the unknown input sequence to the unknown channel based solely on the probabilistic and statistical properties of the input sequence. The receiver can synchronize to the received signal and to adjust the equalizer without the training sequence. The term blind is used in this equalizer because it performs the equalization on the data without a reference signal. Instead, the blind equalizer relies on knowledge of the signal structure and its statistic to perform the equalization. 1. Blind signal is the unknown signal which would be identified in output signal with accommodated noise signal at receiver. 2. Channel equalization uses the idea & knowledge of training sequences for channel estimation where as Blind channel equalization doesn't utilizes the characteristics of training sequences for frequency and impulse response analysis of channel. 3. Blind Channel Equalization differs from channel equalization and without knowing the channel characteristics like transfer function & SNR it efficiently estimate the channel and reduces the ISI by blind signal separation at receiver side by suppressing noise in the received signal.

III.   FRACTIONAL SPACED CMA - CONSTANT MODULUS ALGORITHM

In digital communication, equalizer was designed to compensate the channel distortions, through a process known as equalization. There are two types of equalization which are: 1) Trained equalization, 2) Blind (self-recovering) Equalization.

Blind equalization finds important application in data communication systems. In data communications, digital signals are generated and transmitted by the sender through an analog channel to the receiver. Linear channel distortion as a result of limited channel bandwidth, multipath and fading is often the most serious distortion in digital

communication system. Blind equalization improves system bandwidth efficient by avoiding the use of training sequence.

Channel Equalization:

A typical communication system design involves first passing the signal to be transmitted through a whitening filter to reduce redundancy or correlation and then transmitting the resultant whitened signal. At the receiver, the recorded signal is passed through the inverse whitening filter and the original signal is thus restored. However, the channel will affect the transmitted signal because of a) Channel noise b) Channel dispersion leading to inter symbol interference.

Blind Channel Equalization:

The field of blind channel equalization has been existence for a little over twenty years. Research during this time has centered on developing new algorithms and formulating a theoretical justification for these algorithms. Blind channel equalization is also known as a self-recovering equalization. The objective of blind equalization is to recover the unknown input sequence to the unknown channel based solely on the probabilistic and statistical properties of the input sequence.. A natural question from for direct adaptive equalization with training is,"How can we adapt our filter F, without the use of a training signal". Figure 2 shows such a system. Fractional –spaced CMA used to directly estimate equalizer f . It is similar to CMA. In fractional space it is global convergences.

$$\min J = E\left[(|f^H X(n)|^2 - R_Z)^2 \right] \text{--------(1)}$$

Update rule:

$$f_{n+1} = f_n - \mu . E(|f^H X(n)|^2 - R_Z) X(n) X^H(n) f_n \text{\_\_\_\_(2)}$$

Algorithm:

1. Construct the received sample space.
2. Construct the sample vector $X(n)$.
3. For n=1,2,…….
   Update function (2)
   Calculate instant error
4. Check SER.

IV. NLMS ALGORITHM

Usually, the adaptive algorithm consists of a transfer filter for processing the input single and an algorithm unit for update the transfer filter's coefficients. $x(n)$ is the input signal; $w(n) = [w_0, w_1, w_2, ...w_l]$ is the vector of the transfer filter's coefficients; $d(n)$ is the desired output of the transfer filter; $y(n)$ is the output of the transfer filter; $e(n)$ is the error value, and it can be written as:

$$e(n) = d(n) - \hat{y}(n) \text{-----3}$$
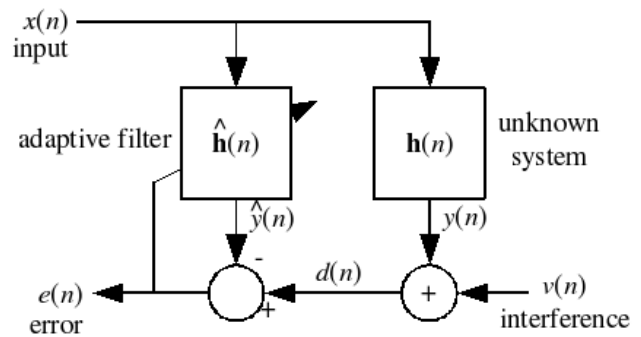
The Adaptive algorithm unit represents some algorithm to update the coefficients of the transfer filter. For LMS algorithm, the method to update the coefficients of the transfer filter is given as follows:

$$w(n) = w(n+1) + \mu^* x(n) * e(n) \text{------4}$$

μ , is the step of LMS algorithm.

The main drawback of the "pure" LMS algorithm is that it is sensitive to the scaling of its input $x(n)$. This makes it very hard (if not impossible) to choose a learning rate μ that guarantees stability of the algorithm. The *Normalised least mean squares filter* (NLMS) is a variant of the LMS algorithm that solves this problem by normalising with the power of the input. The NLMS algorithm can be summarised as:

Parameters: p = filter order μ = step size Initialization:

$$\hat{h}(0) = 0$$

Computation: For $n = 0,1,2,...$

$$X(n) = [x(n), x(n-1), ...x(n-p+1)]^T$$

$$e(n) = d(n) - \overset{\wedge H}{h}(n) X(n)$$

$$\hat{h}(n+1) = \hat{h}(n) + \frac{\mu e^*(n) X(n)}{X^H(n) X(n)}$$

V. ADAPTIVE MMSE EQUALIZER

The Sampled signal after MMSE Equalizer can be expressed in matrix form as

$$\hat{s}(i) = w^H y(i) \text{----5}$$

Where $y(i) = H^T(i) s(i) + n(i)$,--------6

M is the length of the MMSE Equalizer: $w = [w_1, w_2, w_3, w_4, w_5........w_M]^T$ is the equalizer coefficients vector; Then the error signal $e(i)$ is given by $e(i) = d(i) - \hat{s}(i) \text{----7}$

where $d(i)$ is the desired response. For MMSE equalizer, $d(i) = s(i + D)$, D is a time delay parameter which is $L + 1$ usually. The MMSE criterion is used to derive the optimal equalizer coefficients vector w :

$$w = \min imizeE\left\{ |e|^2 \right\} \text{----- } 8$$

We make the assumption that signal $s(i)$ and noise $n(i)$ are independent identity distribution stochastic
Variable and uncorrelated each other, then the equalizer coefficients vector w can be expressed as [2]:

$$w = (H^H H + \frac{1}{SNR} I)^{-1} H^H \delta_D \text{ ----9}$$

Where

$$\delta_D = [0.......1_D;0..........0]^T_{1X(L+M-1)} \qquad SNR = \frac{\sigma_s^2}{\sigma_n^2}$$

denotes the signal noise ratio I is MxM
identity matrix.
To reduce the complexity caused by matrix inversion of ideal MMSE equalizer, we propose an adaptive MMSE equalizer algorithm. In code-multiplexed pilot CDMA systems, conventional adaptive equalizer is difficult to implement for lack of reference signal. In this paper, the steepest descent method[4] is used to derive adaptive equalizer algorithm in code-multiplexed pilot CDMA systems.According to Equation 5 and Equation 7, the mean square error (MSE) J can be expressed as $J(w) = E[e(i)e(i)^*] = \sigma_s^2 - w^H p - p^H w + w^H R w$ ---- 10

where autocorrelation matrix $R = E[y(i)y^H(i)]$; cross-correlation vector $p = E[y(i)d^*(i)], \sigma_s^2$ denotes the

signal power; $(.)^*$ represents conjugate operation. Because the wireless channel is time-varying, the equalizer
coefficients vector w must be updated real time. Conventional adaptive algorithm requires reference signal $d(i)$, while in the downlink of code-multiplexed pilot CDMA systems, $d(i)$ is difficult to distill. To resolve this problem, the steepest decent method is used. From Eqn.8, the gradient vector is

$$\frac{\partial J(w)}{w} = -2p + 2Rw \text{ --- } 11$$

then the equalizer coefficients updating equation is

$$w(i+1) = w(i) + 2\mu[p - Rw(i)] \text{ ----------- } 12$$

where parameter μ is a positive real-valued constant which controls the size of the incremental correction applied to the equalizer coefficients vector.
For the autocorrelation matrix:

$$R = E[y(i)y^H(i)]$$

$$R = E[s(i)s^H(i)]\left\{ H^H(i)H(i) \right\}^T + E[n(i)n^H(i)]$$

$$R = \sigma_s^2 \left\{ H^H(i)H(i) \right\}^T + \sigma_n^2 I$$

---13

the cross-correlation vector

$$p = E[y(i)d^*(i)] = E[(H^T(i)s(i) + n(s))s^*(i - D)]$$

$$p = \sigma_s^2 H^T(i)\delta_D$$

---- 14

From Equations 9,10,11, we can obtain the time recursive equation of MMSE equalizer by:

$$w(i+1) = w(i) + 2\mu\sigma_s^2[H^T(i)\delta_D - (\left\{ H^H(i)H(i) \right\}^T + \frac{1}{SNR} Iw(i)]$$

--------------15

As can be seen from Equation 15, the updating process avoids the matrix inversion operation. On the other hand, the updating process abstains the requirement to store the autocorrelation matrix **R**(i) and only the equalizer coefficients vector of last time is needed. From Equation 15 we know, the channel convolution matrix **H**(i) is required to update the equalizer coefficients vector.
In this paper, the low complexity sliding-window method is used to estimate the channel coefficients, which can be expressed

$$\text{as } \hat{\beta}_l(i) = \frac{1}{2\sqrt{\alpha \, pw(i+1)T_s}} \int_{\tau_l + (i - \frac{w}{2})T_s}^{\tau_l + (i + \frac{w}{2})T_s} y(t)c_p^*(t - \tau_l)dt$$

-------- 16where $\hat{\beta}_l(i)$ is estimation of the complex gain of

l -th path; w is the length of sliding-window in symbols and should be selected properly according to the varying speed of the channel.

VI. EXPERIMENTAL RESULTS

The performances for the Adaptive MMSE and adaptive CMA algorithm through and NLMS algorithm is experimental performed with accurate figures from 1-4.
The performances of the channel estimation is an analyze in such a way that transmitted bits and receiver bits, Equalizers and very important task that is Convergences.
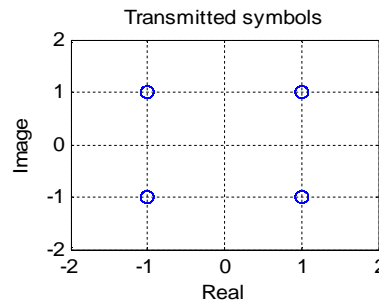

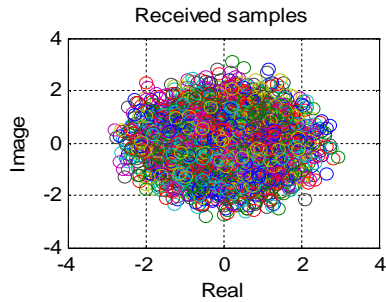Figure 1: Transmitter symbols of Fractional-CMA

Received samples
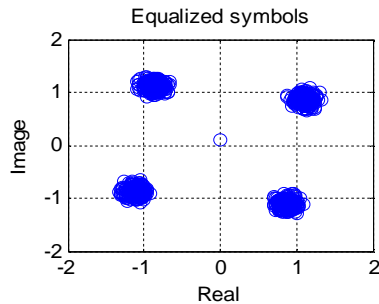
Figure 2: Receiver sample

Equalized symbols

Figure 3: Adaptive Fractional- CMA Equalizer

The first figures from 1-4 are obtained for Adaptive CMA Equalizer, with more efficient for equalization and convergences. Secondly from figure 5-8 are obtained for an Adaptive MMSE equalizer through NLMS algorithm. The efficient of equalization and convergences is too good. The time complexity is very less and more efficient for advance communication systems.
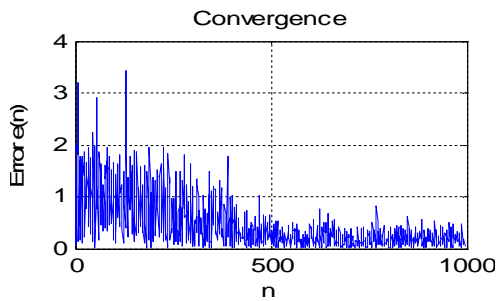
Convergence

Figure 4: Adaptive MMSE Equalizer through NLMS

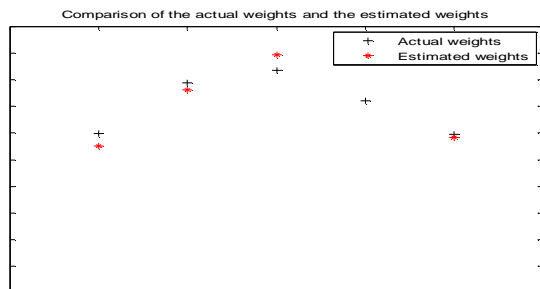## Comparison of the actual weights and the estimated weights

Figure 5: Comparison of the actual weight and estimated weight

VII. CONCLUSIONS

In this paper, Aim at conventional Rake receiver can't satisfy the performance requirement in high data rate transmission, while ideal MMSE equalizer is difficult to real-time implement because its large computational complexity, a low complexity adaptive MMSE equalizer algorithm is proposed. In future conclusion, the proposed low complexity adaptive MMSE equalizer in code-multiplexed CDMA system can be proposed and this system has better practical application value.

REFERENCES

[1] T. P. Krauss, M. D. Zoltowski, and G. Leus : 'Simple MMSE equalizers for CDMA downlink to restore chip sequence : Comparison toZero-Forcing and Rake', ICASSP, May 2000, Vol. 5, pp.2865-2868

[2] K. Hooli, M. Latva-aho, and M. Juntti : 'Multiple access interference suppression with linear chip equalizers in WCDMA downlinkreceivers', GLOBECOM, Dec. 1999, Vol. General Conference (PartA), pp.467-471

[3] L.Mailaender : 'Low-complexity implementation of CDMA downlink equalization. 3G mobilec communication technologies', No.477, pp396~400.

[4] S. Haykin : 'Adaptive Filter Theory', Prentice Hall, third edition, 1996

[5] Gene H. Golub and Charles F. Van Loan :Matrix Computation', TheJohns Hopkins University Press, third edition, 1996

[6] J.Shynk : 'Frequency-domain and multirate adaptive filtering', IEEE Signal Processing Magazine, Vol.9, pp.14-39, Jan.1992

[7] Godard, D. N., "Self-recovering equalization and carrier tracking in two-dimensional data communication systems," IEEE Trans. on Communications.

[8] Fijalkow, I., C. E. Manlove, and C. R. Johnson, Jr., "Adaptive fractionally spaced blind CMA equalization: Excess MSE," IEEE Trans. on Signal Processing, Vol. 46, No. 1, 227–231, 1998.

[9] Ding, Z., R. A. Kennedy, B. D. O. Anderson and C. R. John- son, Jr., "Ill-convergence of godard blind equalizers in data communication systems," IEEE Trans. On Communications, Vol. 39.

[10] Johnson, Jr., C. R., S. Dasgupta, and W. A. Sethares, "Averaging analysis of local stability of a real constant modulus algorithm adaptive filter," IEEE

[11] Brown, D.R., P. B. Schniter, and C. R. Johnson, Jr., "Computationally e cient blind equalization," 35th Annual Allerton Conference on Communication, Control, and Computing, September 1997.

[12] Casas, R. A., C. R. Johnson, Jr., R. A. Kennedy, Z. Ding, and R. Malamut, "Blind adaptive decision feedback equalization: A class of channels resulting in illconvergence from a zero initialization," International Journal on Adaptive Control and Signal Processing Special Issue on Adaptive Channel Equalization.

[13] Johnson, Jr., C. R. and B. D. O. Anderson, "Godard blind equalizer error surface characteristics: White, zeromean, binary source case," International Journal of Adaptive Control and Signal Processing, Vol. 9, 301–324.

[14] http://en.wikipedia.org/nlms.