ISSN 2320 - 2602
Volume 3, No.1, January 2014
International Journal of Advances in Computer Science and Technology
Available Online at http://warse.org/pdfs/2014/ijacst01312014.pdf

A Comparative Study of Different Fuzzy Time Series Forecasting Techniques – Case Study: Marine Fish Production Forecasting

***[1]Vinod K. Yadav, [1]Vidya S.Bharti , [2]M.Krishnan & [2]N.R.Kumar**
[1]Scientist, Central Institute of Fisheries Education, Mumbai 400061, *India*
[2] Principal Scientist, Central Institute of Fisheries Education, Mumbai 400061, *India*
*corresponding author e-mail: vinod_iasri@yahoo.co.in

Abstract: Various forecasting methods have been developed on the basis of fuzzy time series data, but accuracy has been matter of concern in these forecasts. The historical data of marine fish production of India have been taken to implement the model; as such time series data obtained through sample survey are likely to be imprecise. The study uses the fuzzy sets theory of Zadeh [1] and fuzzy time series models introduced by Song and Chissom [2], Chen [3], Chen and Hsu [4] and Singh [5]. The study is aimed to find the marine fish production forecast for a lead year by using different fuzzy time series models. The forecasted marine fish production, obtained through these techniques, have been compared and their performance has been examined and it has been found that forecast obtained by Chen and Hsu[4] is more efficient and provides better forecast in comparison to Singh[5],Chen [3] and Song and Chissom [3] method.

Keywords: Fuzzy Time Series, fuzzy Set, Production, Forecasting, Linguistic Value, fuzzified production. Fuzzy logical relationships.

1. INTRODUCTION

Forecasting plays an important role in our daily life. During the last few decades, various approaches have been developed for time series forecasting. Among them ARIMA models and Box-Jenkins model building approaches are highly rated. But classical time series methods can not deal with forecasting problems in which the values of time series are linguistic terms represented by fuzzy sets Zadeh[1]. Fuzzy time series forecasting emerged as a novel approach for predicting the future values in a situation where neither a trend is viewed nor a pattern in variations of time series are visualized and moreover when information (data) are imprecise and vague. Therefore, Song and Chissom [2] presented the theory of fuzzy time series to overcome this drawback of the classical time series methods. Based on the theory of fuzzy time series, Song *et al.* [2] [6] [10] presented some forecasting methods to forecast the enrollments of the University of Alabama. Chen [3] presented a method to forecast the enrollments of the University of Alabama based on fuzzy time series. It had the advantage of reducing the steps in calculation, time and simplifying the calculation process. Hwang, Chen and Lee [7] used the differences of the enrollments to present a method to forecast the enrollments of the University of Alabama based on fuzzy time series. Huarng[8] used simplified calculations with the addition of heuristic rules to forecast the enrollments using Chen[3]. Chen[9] presented a forecasting method based on high-order fuzzy time series for forecasting the enrollments of the University of Alabama. Chen and Hsu[4] presented a first order time variant method for fuzzy time series for forecasting the enrollments of the University of Alabama.

Singh [5] presented an improved and versatile method for fuzzy time series forecasting using a difference parameter as fuzzy relation for forecasting. All these models have been implemented to forecast the enrollments of the University of Alabama.

In the present paper the different fuzzy time series models introduced by Song and Chissom[2] , Chen[3] , Chen and Hsu[4] and Singh[5] have been implemented on the historical marine fish production forecast, a highly non linear process, where data in general contain imprecision. The study is aimed to get some reliable forecast for marine fish production for a lead year. This production forecast will help the fish farmers as well as the local fish based industries in their business planning.

The rest of this paper is organized as follows. In Section 2, we briefly review the basic concept of fuzzy time series from Song and Chissom [2][6]. In section 3.1, 3.2, 3.3 & 3.4, we obtained the marine fish production forecasting by Song and Chissom[2], Chen [3], Chen and Hsu [4] and Singh [5] model respectively. In section 4 under result and discussion, forecast for Marine fish production by different models are compared. The conclusion are discussed in section 5

2. FUZZY TIME SERIES MODELS

Let $Y(t)$ (t = 0,1,2,..), is a subset of R_1, be the universe of discourse on which fuzzy sets $f_i(t)$ (i = 1,2,…) are defined and $F(t)$ is the collection of f_i (i = 1,2,…). Then $F(t)$ is called fuzzy time series on $Y(t)$ (t = 0,1,2,…). Further $F(t)$ can be understood as a linguistic variable and $f_i(t)$ (i = 1,2,…) as the possible linguistic values of $F(t)$.

Definition 1: Suppose $F(t)$ is caused by a $F(t-1)$ only or by $F(t-1)$ or ($F(t-2)$ or…or $F(t-m)$ ($m > 0$). This relation can be expressed as the following fuzzy relational equation:

$$F(t) = F(t-1) \circ R(t, t-1) \quad \ldots (1)$$

or

$$F(t) = (F(t-1) \cup F(t-2) \cup \& \cup F(t-m)) \circ R_0 \times (t, t-m) \quad \ldots (2)$$

The equation is called the first order model of $F(t)$.

Definition 2: Suppose $F(t)$ is caused by a $F(t-1)$,

$F(t-2), \ldots$, and $F(t-m)$ $(m > 0)$ simultaneously. This relation can be expressed as the following fuzzy relational equation

$$F(t) = (F(t-1) \times F(t-2) \times \ldots \times F(t-m)) \circ R_a(t, t-m) \quad (3)$$

and is called the m^{th} order model of $F(t)$.

Definition 3: If in (1) or (2) or (3), the fuzzy relation $R(t, t-1)$ or $R_a(t, t-m)$ or $R_a(t, t-m)$ of $F(t)$ is dependent of time t, that is to say for different times t_1 and t_2,

$R(t_1, t_1 - 1) = R(t_2, t_2 - 1)$, or $R_a(t_1, t_1 - m) = R_a(t_2, t_2 - m)$ or $R_0(t_1, t_1 - m) = R_0(t_2, t_2 - m)$, then

F (t) is called a time invariant fuzzy time series. Otherwise it is called a time variant fuzzy time series,
In the case of time invariant fuzzy time series,
$R(t, t-1) = R$,
$R_a(t, t-m) = R_a(m)$,

$R_0(t, t-m) = R_0(m)$

In general at different times t_1 and t_2, $R(t_1, t_1 - 1) \neq R(t_2, t_2 - 1)$, $R_a(t_1, t_1 - m) \neq R_a(t_2, t_2 - m)$ and $R^\circ(t_1, t_1 - m) \neq R^\circ(t_2, t_2 - m)$.

There are two reasons for this: first, the universes of discourse on which the fuzzy sets are defined may be different at different times: second the value of $F(t)$ at different times may be different.

Depending upon the complexity of the system, fuzzy time series modeling for a forecast process may use type of relations $R(t, t-1)$, $R_a(t, t-m)$, $R_o(t, t-m)$. Development of fuzzy time series model essentially depends on the procedure of fuzzy relations generated between the observations at a time t and among the observations at different times. Several methods Dubois and Parde (1991), Wu (1986) and Mamdani (1977) are available to determine these relations.

3. MARINE FISH PRODUCTION FORECASTING

3.1. MARINE FISH PRODUCTION FORECASTING BY SONG AND CHISSOM (1993):

Fuzzy time series model deals with situation where the data are linguistic values, in contrast to the conventional time series approaches that typically manipulate numerical data. If data are available in crisp form, it is to be fuzzified before the fuzzy time series methodology can be applied. Fuzzification process starts with defining the universe of discourse U, which contains the historical data and upon which the fuzzy sets are defined.
The study deals with the production of Marine Fish of India (**in lakh Kg**) in various years starting from 1995-96 to 2009-2010 with assumption that it includes some vagueness incurred due to statistical sampling.

Step-1: Let $D_{\min}$ and $D_{\max}$ be minimum and maximum production. Based upon $D_{\min}$ and $D_{\max}$, we define the universe of the discourse U as $[D_{\min} - D_1, D_{\max} + D_2]$, where D_1 and D_2 are two proper positive numbers and accordingly, the universe of discourse U = [26000, 31000]. Further the universe of discourse U is partitioned into five intervals of equal length as follows:

u_1 = [26000, 27000], u_2 = [27000, 28000],
u_3 = [28000, 29000], u_4 = [29000, 30000],
u_5 = [30000, 31000],
Step 2: Fuzzy sets $A_1, A_2, A_3, \ldots A_5$ on universe of discourse, having linguistic values as:
A_1= Poor, A_2 = Average, A_3= good, A_4 = very good A_5 = excellent, are to be defined. $u_1, u_2, \ldots, u_5$ are chosen as elements of these fuzzy sets. The membership grades of $u_1, u_2, \ldots, u_5$ to each A_i (i = 1,2,…, 5) will decide that how well each u_k (k = 1,2,…5) belong to u_i. We have determined the membership of each element in all the fuzzy sets A_i (i = 1,2,…5) and are expressed as

$A_1 = \{u_1/1, u_2/.5, u_3/0, u_4/0, u_5/0\}$
$A_2 = \{u_1/.5, u_2/1, u_3/.5, u_4/0, u_5/0\}$
$A_3 = \{u_1/0, u_2/.5, u_3/1, u_4/.5, u_5/0\}$
$A_4 = \{u_1/0, u_2/0, u_3/.5, u_4/1, u_5/.5\}$
$A_5 = \{u_1/0, u_2/0, u_3/0, u_4/.5, u_5/1\}$
Where u_i (i = 1, 2…5) is the element and the number below '/ ' is the membership of u_i to A_j (j = 1,2,..5)

Step 3: Fuzzify the historical production data to find out the equivalent fuzzy set to each year's production using the step- 2. The equivalent fuzzy set to each year's production are shown in table-1.
Table-1-Fuzzified production for different years

Year	*Actual production (Lakh Kg)*	A_1	A_2	A_3	A_4	A_5	*Fuzzified Production*
1995-96	27070	.5	1	.5	0	0	A_2
1996-97	29670	0	0	.5	1	.5	A_4
1997-98	29500	0	0	.5	1	.5	A_4
1998-99	26960	1	.5	0	0	0	A_1
1999-2000	28520	0	.5	1	.5	0	A_3
2000-2001	28110	0	.5	1	.5	0	A_3
2001-2002	28300	0	.5	1	.5	0	A_3
2002-2003	29900	0	0	.5	1	.5	A_4
2003-2004	29410	0	0	.5	1	.5	A_4
2004-2005	27800	.5	1	.5	0	0	A_2
2005-2006	28160	0	.5	1	.5	0	A_3
2006-2007	30240	0	0	0	.5	1	A_5
2007-2008	29200	0	0	.5	1	.5	A_4
2008-2009	30000	0	0	0	.5	1	A_5
2009-2010	29800	0	0	.5	1	.5	

Step 4: Fuzzy logical relationship of the production have been obtained from Table-1, where the fuzzy logical relationship $A_j \rightarrow A_k$ means: if the production of year j is A_j then that of year j + 1 is A, where A_j is called the current state of production and A_k is called the next state of the production. The fuzzy logical relationship and group for production is derived in Table-2 and Table-3:

Table- 2-Fuzzy logical relationship of production

$A_2 \to A_4$	$A_4 \to A_4$	$A_4 \to A_1$	$A_1 \to A_3$	$A_3 \to A_3$
$A_3 \to A_3$	$A_3 \to A_4$	$A_4 \to A_4$	$A_4 \to A_2$	$A_2 \to A_3$
$A_3 \to A_5$	$A_5 \to A_4$	$A_4 \to A_5$		

Table-3- Fuzzy logical relationship groups for production
Based on the Fuzzy logical relationship, we derive the fuzzy logical relationship groups for production, which comes to be:
Group-1: $A_1 \to A_3$
Group-2: $A_2 \to A_4, A_3,$
Group-3: $A_3 \to A_3, A_4, A_5$
Group-4: $A_4 \to A_4, A_1, A_2, A_5$
Group-5: $A_5 \to A_4$

Step 5. Based on fuzzy logical relational groups a total of 11 relations $R_1,\ldots..R_{10}$ are to be computed. If fuzzy logical relation is $A_i \to A_j$ then the time invariant relation $R_{ij} = A_i^T \times A_j$, the elements of the matrix R_{ij} be computed as $d_{ij} = \min(A_i^T, A_j)$ (i,j = 1,…m)

Here,
for $A_1 \to A_3$: (R_{13}) $R_1 = A_1^T \times A_3$
for $A_2 \to A_4$: (R_{24}) $R_2 = A_2^T \times A_4$
for $A_2 \to A_3$: (R_{23}) $R_3 = A_2^T \times A_3$
for $A_3 \to A_3$: (R_{33}) $R_4 = A_3^T \times A_3$
for $A_3 \to A_4$: (R_{34}) $R_5 = A_3^T \times A_4$
for $A_3 \to A_5$: (R_{35}) $R_6 = A_3^T \times A_5$
for $A_4 \to A_4$: (R_{44}) $R_7 = A_4^T \times A_4$
for $A_4 \to A_1$: (R_{41}) $R_8 = A_4^T \times A_1$
for $A_4 \to A_2$: (R_{42}) $R_9 = A_4^T \times A_2$
for $A_4 \to A_5$: (R_{45}) $R_{10} = A_4^T \times A_5$
for $A_5 \to A_4$: (R_{54}) $R_{11} = A_5^T \times A_4$

The first order fuzzy time invariant relation R is computed as $R(t, t\text{-}1) = R = \bigcup_{1}^{11} R_i$, $\cup$ is the union operator as defined earlier(max).Taking the union of all 11 fuzzy relational matrices, the obtained relation

$$R = \begin{bmatrix} 0 & .5 & 1 & .5 & 0 \\ 0 & .5 & 1 & 1 & .5 \\ .5 & .5 & 1 & 1 & 1 \\ 1 & 1 & .5 & 1 & 1 \\ .5 & .5 & .5 & 1 & .5 \end{bmatrix}$$

Step 6 The Computation of fuzzy output is carried out by the forecasting model

$$A_i = A_{i-1} \circ R$$

Here, A_{i-1} is the fuzzified production of the year i-1 (known as current state) and A_i the fuzzified forecasted production of the year i. and 'o' is the max-min operator. Thus the forecasted values for the year 1996-97 to 2008-2009 can be computed. The fuzzified output can be computed and are obtained as Table no. 4

Table No. - 4-Fuzzified output

Year	*Actual production (Lakh Kg)*	A_1	A_2	A_3	A_4	A_5
1996-97	29670	.5	.5	1	1	.5
1997-98	29500	1	1	.5	1	1
1998-99	26960	1	1	.5	1	1
1999-2000	28520	0	.5	1	.5	.5
2000-2001	28110	.5	.5	1	1	1
2001-2002	28300	.5	.5	1	1	1
2002-2003	29900	.5	.5	1	1	1
2003-2004	29410	1	1	.5	1	1
2004-2005	27800	1	1	.5	1	1
2005-2006	28160	.5	.5	1	1	.5
2006-2007	30240	.5	.5	1	1	1
2007-2008	29200	.5	.5	.5	1	.5
2008-2009	30000	1	1	.5	1	1

Step 7 The forecasted values in the table 4 are to be defuzzified to get the crisp output. Several defuzzification methods are available. The easiest way to calculate the crisp output is with the following rule:

1. If the membership of an output has only one maximum, the select the midpoint of that corresponding interval as forecasted value.
2. If the membership of an output has two or more consecutive maximum, the select the midpoint of the corresponding conjunct intervals as forecasted value.
3. Otherwise, standardize the fuzzy output and use the midpoint of each interval to calculate the centroid of the fuzzy set as forecasted value.

The forecasted values thus obtained along with the actual production are placed in table 5

3.2. MARINE FISH PRODUCTION FORECASTING BY CHEN (1996):

Chen (1996) gave a simplified approach of using aromatic operations in place of calculating the relational matrices and then applying the min-max composition operations for getting the fuzzified forecasted values and then applying the defuzzification method.

Computational procedure:

Continue the steps mentioned in the above method upto step 4.then

1) If the production of the year i is A_j and fuzzy logical relation is $A_j \rightarrow A_k$ and A_k has max membership in interval u_k, then the forecasted production for the year i=i+1 will be midpoint of A_k.
2) If the fuzzified production of the year i is A_j and there are fuzzy logical relationships in the fuzzy logical relationship group as:

$$A_j \rightarrow A_{k1},\ A_j \rightarrow A_{k1}, \ldots\ A_j \rightarrow A_{kp}$$

A_{k1}, A_{k1}, ….. , A_{kp} has max membership in the intervals $u_{k1}, u_{k2}, \ldots, u_{kp}$ respectively and $m_1, m_2, \ldots m_p$ are their respective midpoints , then the forecasted production for the year i+1 will be ($m_1, m_2, \ldots m_p$)/p .If the fuzzified production of a year *i* is A_j, and no logical relationship is found in logical relationship groups, whose current state of production is A_j ,where the maximum membership value of A_j occurs at interval u_j and the midpoint of u_j. is m_j then the forecasted production of year $i + 1$ is m_j. The forecasted values thus obtained along with the actual production are placed in table 6

Table-5-Forecasted production along with actual production by Song and Chissom (1993)

Year	*Actual production (Lakh Kg)*	*Forecasted Production(Lakh Kg)*
1995-96	27070	
1996-97	29670	29000
1997-98	29500	28500
1998-99	26960	28500
1999-2000	28520	28500
2000-2001	28110	29500
2001-2002	28300	29500
2002-2003	29900	29500
2003-2004	29410	28500
2004-2005	27800	28500
2005-2006	28160	29000
2006-2007	30240	30000
2007-2008	29200	29500
2008-2009	30000	28500

Table-6 Forecasted production along with actual production by Chen (1996)

Year	*Actual production (Lakh Kg)*	*Forecasted Production (Lakh Kg)*
1995-96	27070	
1996-97	29670	29000
1997-98	29500	28500
1998-99	26960	28500
1999-2000	28520	28500
2000-2001	28110	30000
2001-2002	28300	30000
2002-2003	29900	30000
2003-2004	29410	28500
2004-2005	27800	28500
2005-2006	28160	29000
2006-2007	30240	29500
2007-2008	29200	29500
2008-2009	30000	28500

3.3. MARINE FISH PRODUCTION FORECASTING BY CHEN AND HSU (2004) METHOD

Chen and Hsu (2004) method defines the universe of discourse and partitions the universe of discourse into some even and equal length intervals. Then, it gets the statistical distributions of the historical production data in each interval and re-divided each interval. Later it defines linguistic values represented by fuzzy sets based on the re-divided intervals and fuzzify the historical production to get fuzzified production. Then, it establishes fuzzy logical relationships based on the fuzzified production. Finally, it uses a set of rules to determine the direction of change in the trend of the forecasting whether it registers an upward or downward and then to forecast the production.

Assume that we want to forecast the production of year n, then the "difference of differences" of the production between years n-1 and n-2 and between years n-2 and n-3 =(the production of year n-1 - the production of year n-2) - (the production of year n-2 – the production of year n-3). The method is now presented as follows:

Step 1: Define the universe of discourse U and partition it into several even and equal length intervals $u_1, u_2, \ldots$, and u_n. For example, assume that the universe of discourse U＝[26000, 31000] is partitioned into five even and equal length intervals u_1, u_2, u_3, u_4 and u_5, where u_1 ＝ [26000, 27000], u_2 ＝[27000, 28000], u_3＝[28000, 29000], u_4＝[29000, 30000], u_5＝[30000, 31000

Step 2: Get a statistics of the distribution of the historical production in each interval. Sort the intervals based on the number of historical production data in each interval from the highest to the lowest. Find the interval having the largest number of historical production data and divide it into four sub-intervals of equal length. Find the interval having the second largest number of historical production data and divide it into three sub-intervals of equal length. Find the interval having the third largest number of historical production data and divide it into two sub-intervals of equal length. Find the interval with the fourth largest number of historical production data and let the length of this interval remain unchanged. If there are no data distributed in an interval, and then discard this interval. For example, the distributions of the historical production data in different intervals are summarized as shown in Table 7

Table 7. The distribution of the historical fish production data

Intervals	26000-27000	27000-28000	28000-29000	29000-30000	30000-31000
Number of historical production data	1	2	4	6	2

After executing this step, the universe of discourse [26000, 31000] is re-divided into the following intervals:

u_1＝ [26000,27000],	$u_{2,1}$ ＝[27000,27500],
$u_{2,2}$ ＝[27500,28000],	$u_{3,1}$＝[28000,28333],
$u_{3,2}$＝[28333,28667],	$u_{3,3}$＝[28667,29000],
$u_{4,1}$＝[29000,29250],	$u_{4,2}$＝[29250, 29500],
$u_{4,3}$＝[29500, 29750],	$u_{4,4}$＝[29750,30000],

$u_{5,1}$=[30000,30500],	$u_{5,2}$ =[30500,31000]

Step 3: Define each fuzzy set Ai based on the re-divided intervals and fuzzify the historical production shown in Table 1, where fuzzy set A*i* denotes a linguistic value of the production represented by a fuzzy set, and $1 \le i \le 12$. For example, A_1=very very very very few, A_2=very very very few, A_3=veryvery few, A_4 = very few, A_5 = few, A_6 =moderate, A_7=many, A_8=many many, A_9=very many, A_{10}=too many, A_{11}=too many many and A_{12}=too many many many defined as follows

$A_1=1/u_1+0.5/u_{2,1}+0/u_{2,2}+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0/u_{4,4}+0/u_{5,1}+0/u_{5,2}$,

$A_2=0.5/u_1+1/u_{2,1}+0.5/u_{2,2}+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0/u_{4,4}+0/u_{5,1}+0/u_{5,2}$

$A_3=0/u_1+0.5/u_{2,1}+1/u_{2,2}+0.5/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0/u_{4,4}+0/u_{5,1}+0/u_{5,2}$

$A_4=0/u_1+0/u_{2,1}+0.5/u_{2,2}+1/u_{3,1}+0.5/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0/u_{4,4}+0/u_{5,1}+0/u_{5,2}$

$A_5=0/u_1+0/u_{2,1}+0/u_{2,2}+0.5/u_{3,1}+1/u_{3,2}+0.5/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0/u_{4,4}+0/u_{5,1}+0/u_{5,2}$

$A_6=0/u_1+0/u_{2,1}+0/u_{2,2}+0/u_{3,1}+0.5/u_{3,2}+1/u_{3,3}+0.5/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0/u_{4,4}+0/u_{5,1}+0/u_{5,2}$,

$A_7=0/u_1+0/u_{2,1}+0/u_{2,2}+0/u_{3,1}+0/u_{3,2}+0.5/u_{3,3}+1/u_{4,1}+0.5/u_{4,2}+0/u_{4,3}+0/u_{4,4}+0/u_{5,1}+0/u_{5,2}$,

$A_8=0/u_1+0/u_{2,1}+0/u_{2,2}+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0.5/u_{4,1}+1/u_{4,2}+0.5/u_{4,3}+0/u_{4,4}+0/u_{5,1}+0/u_{5,2}$,

$A_9=0/u_1+0/u_{2,1}+0/u_{2,2}+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0.5/u_{4,2}+1/u_{4,3}+0.5/u_{4,4}+0/u_{5,1}+0/u_{5,2}$,

$A_{10}=0/u_1+0/u_{2,1}+0/u_{2,2}+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0.5/u_{4,3}+1/u_{4,4}+0.5/u_{5,1}+0/u_{5,2}$,

$A_{11}=0/u_1+0/u_{2,1}+0/u_{2,2}+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0.5/u_{4,4}+1/u_{5,1}+0.5/u_{5,2}$,

$A_{12}=0/u_1+0/u_{2,1}+0/u_{2,2}+0/u_{3,1}+0/u_{3,2}+0/u_{3,3}+0/u_{4,1}+0/u_{4,2}+0/u_{4,3}+0/u_{4,4}+0.5/u_{5,1}+1/u_{5,2}$

For simplicity, the membership values of fuzzy set A_i either are 0, 0.5 or 1, where$1 \le i \le 12$. Then, fuzzify the historical production shown in Table 2 and the linguistic values of the production A_1, A_2, …, A_{12}. The reason for fuzzifying the historical production into fuzzified production is to translate crisp values into fuzzy sets to get a fuzzy time series (Table-8).

Table-8. Fuzzified output

Year	*Actual production (Lakh Kg)*	*Fuzzified Production*
1995-96	27070	A_2
1995-96	29670	A_9
1997-98	29500	A_8
1998-99	26960	A_1
1999-2000	28520	A_5
2000-2001	28110	A_4
2001-2002	28300	A_4
2002-2003	29900	A_{10}
2003-2004	29410	A_8
2004-2005	27800	A_3
2005-2006	28160	A_4
2006-2007	30240	A_{11}
2007-2008	29200	A_7
2008-2009	30000	A_{10}
2009-2010	29800	A_{10}

Step 4: Establish fuzzy logical relationships based on the fuzzified production:
Where the fuzzy logical relationship " $A_i \to A_j$ " denotes " if the fuzzified production of year n-1 is A_i , then the fuzzified production of year n is A_j ". For example, based on the fuzzify historical production obtained in Step 3, we can get the fuzzy logical relationships as shown in Table 9.

Table 9. Fuzzy logical relationships

$A_2 \to A_9$	$A_9 \to A_8$
$A_8 \to A_1$	$A_1 \to A_5$
$A_5 \to A_4$	$A_4 \to A_4$
$A_4 \to A_{10}$	$A_{10} \to A_8$
$A_8 \to A_3$	$A_3 \to A_4$
$A_4 \to A_{11}$	$A_{11} \to A_7$
$A_7 \to A_{10}$	$A_{10} \to A_{10}$

Step 5: Divide each interval derived in Step 2 into four subintervals of equal length, where the 0.25-point and 0.75-point of each interval are used as the upward and downward forecasting points of the forecasting. Use the following rules to determine whether the trend of the forecasting goes up or down and to forecast the production. Assume that the fuzzy logical relationship is $A_i \to A_j$, where A_i denotes the fuzzified production of year n-1 and A_j denotes the fuzzified production of year n, then (1) If $j > i$ and the difference of the differences of the production between years n-1 and n-2 and between years n-2 and n-3 is positive, then the trend of the forecasting will go up, and we use the following **Rule 2** to forecast the production; (2) If $j > i$ and the difference of the differences of the production between years n-1 and n-2 and between years n-2 and n-3 is negative, then the trend of the forecasting will go down, and we use the following **Rule 3** to forecast the production;(3) If $j < i$ and the difference of the differences of the production between years n-1 and n-2 and between years n-2 and n-3 is positive, then the trend of the forecasting will go up, and we use the following **Rule 2** to

forecast the production; (4) If j < i and the difference of the differences of the production between years n-1 and n-2 and between years n-2 and n-3 is negative, then the trend of the forecasting will go down, and we use the following

Rule 3 to forecast the production; (5) If j = i and the difference of the differences of the production between years n-1 and n-2 and between years n-2 and n-3 is positive, then the trend of the forecasting will go up, and we use the following **Rule 2** to forecast the production;
(6) If j = i and the difference of the differences of the production between years n-1 and n-2 and between years n-2 and n-3 is negative, then the trend of the forecasting will go down, and we use the following **Rule 3** to forecast the production, where **Rule 1**, **Rule 2** and **Rule 3** are given below:

Rule 1: When forecasting the production of year 1997-98, there are no data before the production of year 1995-96, therefore we are not able to calculate the difference of the production between years 1995-96 and 1994-95 and the difference of the differences between years 1996-97 and 1995-96 and between years 1995-96 and 1994-95. Therefore, if |(the difference of the production between years 1996-97 and 1995-96)|/2 >half of the length of the interval corresponding to the fuzzified production Aj with the membership value equal to 1, then the trend of the forecasting of this interval will be upward, and the forecasting production falls at the 0.75-point of this interval; if |(the difference of the production between years 1996-97 and 1995-96)|/2 = half of the length of the interval corresponding to the fuzzified production A_j with the membership value equal to 1, then the forecasting production falls at the middle value of this interval; if |(the difference of the production between years 1996-97 and 1995-96)|/2 < half of the length of the interval corresponding to the fuzzified production Aj with the membership value equal to 1, then the trend of the forecasting of this interval will be downward, and the forecasting production falls at the 0.25-point of the interval.

Rule 2: If (|the difference of the differences between years n-1 and n-2 and between years n-2 and n-3| × 2 + the production of year n-1) or (the production of year n-1 - |the difference of the differences between years n-1 and n-2 and between years n-2 and n-3| × 2) falls in the interval corresponding to the fuzzified production Aj with the membership value equal to 1, then the trend of the forecasting of this interval will be upward, and the forecasting production falls at the 0.75-point of the interval of the corresponding fuzzified production Aj with the membership value equal to 1; if (|the difference of the differences between years n-1 and n-2 and between years n-2 and n-3|/2+ the production of year n-1) or (the production of year n-1 - |the difference of the differences between years n-1 and n-2 and between years n-2 and n-3|/2) falls in the interval of the corresponding fuzzified production Aj with the membership value equal to 1, then the trend of the forecasting of this interval will be downward, and the forecasting value falls at the 0.25-point of the interval of the corresponding fuzzified production Aj with the membership value equal to 1; if neither is the case, then we let the forecasting production be the middle value of the interval corresponding to the fuzzified production Aj with the membership value equal to 1.

Rule 3: If (|the difference of the differences between years n-1 and n-2 and between years n-2 and
n-3|/2 + the production of year n-1) or (the production of year n-1 - |the difference of the differences between years n-1 and n-2 and between years n-2 and n-3|/2) falls in the interval of the corresponding fuzzified production Aj with the membership value equal to 1, then the trend of the forecasting of this interval will be downward, and the forecasting production falls at the 0.25-point of the interval corresponding to the fuzzified production A_j with the membership value equal to 1; if (|the difference of the differences between years n-1 and n-2 and between years n-2 and n-3| × 2 + the production of year n-1) or (the production of year n-1 - |the difference of the differences between years n-1 and n-2 and between years n-2 and n-3| × 2) falls in the interval corresponding to the fuzzified production Aj with the membership value equal to 1, then the trend of the forecasting of this interval will be upward, and the forecasting production falls at the 0.75-point of the interval corresponding to the fuzzified production A_j with the membership value equal to 1; if neither is the case, then we let the forecasting production be the middle value of the interval corresponding to the fuzzified production A_j with the membership value equal to 1.

Table 10 summarizes the forecasting results of the Chen and Hsu(2004) method from 1996-97 to 2009-10, where the universe of discourse is divided into 12 intervals and the interval with the largest number of historical enrollment data is divided into 4 sub-intervals of equal length.

Table-10. Forecasted production along with actual production by Chen and Hsu (2004)

Year	*Actual production (Lakh Kg)*	*Forecasting production(Lakh Kg)*
1995-96	27070	
1996-97	29670	29613
1997-98	29500	29438
1998-99	26960	26500
1999-2000	28520	28500
2000-2001	28110	28167
2001-2002	28300	28167
2002-2003	29900	29875
2003-2004	29410	29375
2004-2005	27800	27750
2005-2006	28160	28167
2006-2007	30240	30250
2007-2008	29200	29125
2008-2009	30000	29875
2009-2010	29800	29875

3.4. MARINE FISH PRODUCTION FORECASTING BY SINGH (2007)

RULES FOR FORECASTING

Some notations used are defined as

$[*A_j]$ is corresponding interval u_j for which membership in A_j is Supremum (ie 1).

$L[*A_j]$ is the lower bound of interval u_j

$U[*A_j]$ is the upper bound of interval u_j

$l[*A_j]$ is the length of the interval u_j whose membership in A_j is Supremum (ie 1)

$M[*A_j]$ is the mid value of the interval u_j having Supremum value in A_j

For a Fuzzy logical relation $A_i \rightarrow A_j$:

A_i is the fuzzified production of year n

A_j is the fuzzified production of year n+1

E_i is the actual productionof year n

E_{i-1} is the actual production of year n-1

E_{i-2} is the actual production of year n-2

F_j is the crisp forecasted production of the year n+1

This Model of order three utilizes the historical data of year n-2, n-1, n for framing rules to implement on fuzzy logical relation, $A_i \rightarrow A_j$, where A_i, the current state, is the fuzzified production of year n and A_j , the next state, is fuzzified production of year n+1. The proposed method for forecasting is mentioned as Rule for generating the relations between the time series data of years n-2, n-1, n for forecasting the enrollment of year n+1.

Computational Algorithm: (Forecasting production F_j for year n+1(ie 1998-1999) and onwards.

For k=3 to …….K (end of time series data)

Obtained fuzzy logical Relation for year k to k+1

$A_i \rightarrow A_j$

R= 0 and S= 0

Compute

$D_i = \Big| \, |(E_i - E_{i-1})| - |(E_{i-1} - E_{i-2})| \, \Big|$

$X_i = E_i + D_i/2$

$XX_i = E_i - D_i/2$

$Y_i = E_i + D_i$

$YY_i = E_i - D_i$

$Pi = E_i + D_i/4$

$P_{ii} = E_i - D_i/4$

$Q_i = E_i + 2*D_i$

$Q_{ii} = E_i - 2*Di$

If $X_i \geq L[*A_j]$ And $X_i \leq U[*A_j]$

Then $R = R + X_i$ And $S = S + 1$

If $X X_i \geq L[*A_j]$ And $XX_i \leq U[*A_j]$

Then $R = R + XX_i$ and $S = S + 1$

If $Y_i \geq L[*A_j]$ And $Y_i \leq U[*A_j]$

Then $R = R + Y_i$ and $S = S +1$

If $YY_i \geq L[*A_j]$ And $YY_i \leq U[*A_j]$

Then $R = R + YY_i$ And $S = S +1$

If $P_i \geq L[*A_j]$ And $P_i \leq U[*A_j]$

Then $R = R + P_i$ And $S = S + 1$

If $PP_i \geq L[*A_j]$ And $PP_i \leq U[*A_j]$

Then $R = R + PP_i$ and $S = S + 1$

If $Q_i \geq L[*A_j]$ And $Q_i \leq U[*A_j]$

Then $R = R + Q_i$ and $S = S +1$

If $QQ_i \geq L[*A_j]$ And $QQ_i \leq U[*A_j]$

Then $R = R + QQ_i$ And $S = S +1$

$F_j = (R+ M(*A_j))/(S +1)$

Next k

4. Results & Discussion

In time series forecasting, the forecasting accuracy of a model is commonly measured in terms of Mean Square Error (MSE) or in terms of Average Error. Lower the MSE or average error, better the forecasting method. MSE is defined as

Mean Square Error =

$$\frac{\sum_{i=1}^{n}(actual\ value_i - forecasted\ value_i)^2}{n}$$

and forecasting error as

Forecasting error (in percent) =

$$\frac{|\,forecasted - actaul\ value\,|}{actual\ value} \times 100$$

Average forecasting error (in percent)

$$= \frac{sum\ of\ forecasting\ error}{numbers\ of\ errors}$$

Using the above algorithms, (Rule for forecasting), implemented through BASE SAS -programming language, the computations have been carried out with the model and the results obtained are placed in the table-11 along with results of other models:

With the above comparison of actual production of marine fish production of India with the forecasted production by Chen and Hsu[4], one can conclude that the forecasted results are very close to that of actual result (fig.1). The MSE and Average Error(%) of the forecasting results of the Chen and Hsu[4] method is smaller than that of Singh[5],Song and Chissom [2] & Chen [3] methods(table-11). Hence, the Chen and Hsu [4] method can get a higher forecasting accuracy rate for forecasting marine fish production of India than the Singh [5], Song and Chissom [2] & Chen[3] other methods studied.

5. CONCLUSIONS

The motivation of using different fuzzy time series model is that the historical marine fish production data are collected through various sampling techniques involving the vagueness. In this paper, we have presented different methods for forecasting the marine fish production in India using fuzzy time series. The Chen and Hsu [5], method belongs to the first order and time-variant methods. From Table-11, we can see that the MSE of the forecasting results of this method is smaller than that of the other three methods. That is, the first order and time-variant methods gets a higher forecasting accuracy rate for forecasting fish production than the other three said methods.

Acknowledgement: The authors are grateful to the Director CIFE, Mumbai for encouragement and providing the facilities under the Institutional project (Project code: CIFE-2011/9)

REFERENCES

[1] Zadeh L A, Fuzzy sets. *Information and Control*, 8 (1965) 338-353.

[2] Song Q & Chissom B S, Fuzzy time series and its models. *Fuzzy Sets and Systems*, 54 (1993).

[3] Chen S M, Forecasting enrollments based on fuzzy time series, Fuzzy *Sets and Systems*, 81 (1996) 311-319.

[4] Chen S. M. and Hsu C.C. (2004). A new method to forecasting production using fuzzy time series.*International Journal of Applied Science and Engineering*, 2(3), 234-244.

[5] Singh S R, A Simple method of forecasting based on fuzzy time series, Applied *mathematics and computation*, 186 (2007) 330-339.

[6] Song, Q. and Chissom, B. S. (1994).Forecasting enrollments with fuzzy time series - Part II. *Fuzzy Sets and Systems*, **62**, 1-8

[7] Hwang J R, Chen S M & Lee C H, Handling forecasting problems using fuzzy time series. *Fuzzy Sets and Systems*, 100 (1998) 217-228

[8] Huarng K, Heuristic models of fuzzy time series for forecasting. *Fuzzy Sets and Systems*, 123 (2001) 369-386

[9] Chen S M, Forecasting enrollments based on high-order fuzzy time series, *Cybernetics and Systems: An International Journal*, 33 (2002) 1-16.ng LST images obtained from the ANN model are presented in the Figures 6 to 9.

[10] Song Q, A note on fuzzy time series model selection with sample autocorrelation functions. *Cybernetics and Systems: An International Journal*, 34 (1996) 93-107.

[11] Chen S M & Hwang J R ,Temperature prediction using fuzzy time series, *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, 30 (2000) 263-275.

Table 11- A comparison of the forecasting results of different forecasting methods

Year	*Actual production (Lakh Kg)*	*Forecasted Production by Singh(2007)*	*Forecasted Production by* Song and Chissom (1993) ,	*Forecasted Production* by Chen (1996)	Forecasted Production by Chen and Hsu(2004)
1995-96	27070				
1996-97	29670		29000	29000	29613
1997-98	29500		28500	28500	29438
1998-99	26960	26500.00	28500	28500	26500
1999-2000	28520	28322.50	28500	28500	28500
2000-2001	28110	28392.50	29500	30000	28167
2001-2002	28300	28527.50	29500	30000	28167
2002-2003	29900	29500.00	29500	30000	29875
2003-2004	29410	29414.16	28500	28500	29375
2004-2005	27800	27345.00	28500	28500	27750
2005-2006	28160	28465.00	29000	29000	28167
2006-2007	30240	30580.00	30000	29500	30250
2007-2008	29200	29563.30	29500	29500	29125
2008-2009	30000	30370.00	28500	28500	29875
2009-2010	29800	29720.00	29500	29500	29875
Mean Square Error(MSE)		**102760.25**	**867950**	**1153783**	**19383.07**
Average Error(%)		**1.01**	**2.72**	**3.07**	**0.3**

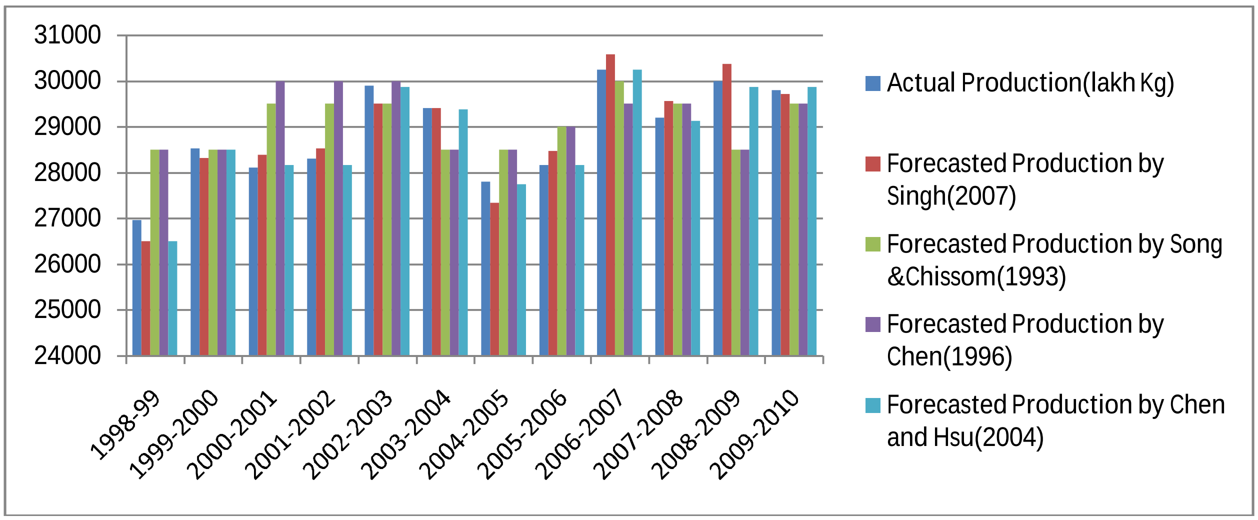

Figure 1: Actual Marine Fish Production vs. forecasted Marine Fish Production