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Filtering using lowest measurable frequencies

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ABSTRACT

In general the Z-transform of any function written explicitly can be found for a set of frequencies in a certain domain at our choice. Applying any of the filters (low pass, high pass, band pass,…) . we use the Z-transform for specified intervals. Then we take the inverse Z-transform to get the required filtered image. The results are promising and make you think of using different filters or different transform. We set the z-transform at some specified frequencies, which are connected the grid distance used in the image display.

Keywords
Z-transform, Z-domain, Frequency domain, Grid distance.

1. INTRODUCTION

The use of z-transform simplifies the evaluation of difference equations, consequently it is widely used in the analysis and synthesis of discrete systems. the z transform will convert a real discrete-time signal into a function of a complex variable z.
The z-transform is useful for the manipulation of discrete data sequences and has acquired a new significance in the formulation and analysis of discrete-time systems. It is used extensively today in the areas of applied mathematics, digital signal processing, control theory, population science, economics. These discrete models are solved with difference equations in a manner that is analogous to solving continuous models with differential equations[9].
either in the time domain or in the frequency domain. Time-domain and frequency domain representation methods offer alternative insights into a system, and depending on the application it may be more convenient to use one method in preference to the other. Time domain system analysis methods are based on differential equations which describe the system output as a weighted combination of the differentials (i.e. the rates of change) of the system input and output signals[3]. The description of a system in the frequency domain can reveal valuable insight into the system behaviour and stability[4].
System analysis in frequency domain can also be more convenient as differentiation and integration operations are performed through multiplication and division by the frequency variable respectively. Furthermore the transient and the steady state characteristics of a system can be predicted by analysing the roots of the Laplace transform or the z-transform, the so-called poles and zeros of a system. A special feature of the z-transform is that for the signals and system of interest to us , all of the analysis will be in terms of ratios of polynomial. working with these polynomials is relatively straight forward.

2. RELATED WORKS

There are two methods for smoothing a sequence of numbers in order to approximate a low-pass filter: the polynomial fit, as just described, and the moving average.
In the first case, the approximation to a LPF can be improved by using a higher-degree polynomial: for example, instead of using a quadratic as in the example given in the previous lecture, we could have fitted a least-squares quartic to the original “noisy” data. The effect of using a higher-degree polynomial is to give both a higher degree of tangency at ω_j =0and a sharper cut-off in the amplitude response.[10]

In general a filter takes an input **x** and produces an output **y**:
$\mathbf{x} \rightarrow \mathbf{y}$
Usually a filter is specified in terms of some frequency response, say **C[**ω_j**]** which we apply to a time series x_k. As already mentioned, we can apply the effects of the

filter in either the time domain or the frequency domain.
In the time domain, we just convolve $\boldsymbol{x_k}$ with the inverse Fourier transform of **C[** ω_j]. In the frequency domain we multiply **C[** ω_j] = ***Cj*** and the Fourier transform of $\boldsymbol{x_k}$.. [12]

2. DEFINITION OF THE Z-TRANFORM

Given a finite length $x[n]$, the z-tranform is defined as

$$X(z)=\sum_{k=0}^{N}x[k]z^{-k}=\sum_{k=0}^{N}x[k](z^{-1})^{k}$$

Where the sequence support interval is [0,N], and z is any complex number, This transformation produces a new representation of $x[n]$ denoted $X(z)$,Returning to the original sequence $x[n]$ requires finding the coefficient associated with the n th per of z $^{-1}$ [10]

Formally transforming from the time/sequence/ n -domain to the z-domain is represented as [4]

n -Domain $\overset{z}{\longleftrightarrow}$ z-Domain

$$x[n]=\sum_{k=0}^{N}x[k]\delta[n-k]\overset{z}{\longleftrightarrow}X(z)=\sum_{k=0}^{N}x[k]z^{-k}$$

(2)

A sequence and its z-transform are said to form a z-transform pair and are denoted

$$x[n]\overset{z}{\longleftrightarrow}X(z) \qquad (3)$$

In the z-domain the independent variable is z

3.PROPERTIES OF THE Z-TRANSFORM

The z-transform has a few very useful propertied, and its definition extends to finite signals/impulse response[1]

3.1 THE SUPERPOSITION (LINERATY) PROPERTY

$$ax_1[n]+bx_2[n]\overset{z}{\longleftrightarrow}aX_1(z)+bX_2(z) \qquad (4)$$

Proof

$$X(z)=\sum_{n=0}(ax_1[n]+bx_2[n])z^{-1}$$

$$=a\sum_{n=0}^{N}x_1[n]z^{-1}+b\sum_{n=0}^{N}x_2[n]z^{-1}$$

$$=aX_1(z)+bX_2(z)$$

3.2 THE DELAY PROPERTY

$$x[n-1]\overset{z}{\longleftrightarrow}z^{-1}X(z) \qquad (5)$$

And

$$x[n-n_0]\overset{z}{\longleftrightarrow}z^{-n_0}X(z) \qquad (6)$$

Proof: consider

(1)

$$X(z)=\alpha_0+\alpha_1z^{-1}+\ldots+\alpha_Nz^{-N}$$

Then

$$x[n]=\sum_{k=0}^{N}\alpha_k\delta[n-k]$$

$$=\alpha_0\delta[n]+\alpha_1\delta[n-1]+\ldots+\alpha_N\delta[n-N]$$

Let

$$Y(z)=z^{-1}X(z)$$

$$\alpha_0z^{-1}+\alpha_1z^{-2}+\ldots+\alpha_Nz^{-N-1}$$

So

$$y[n]=\alpha_0\delta[n-1]+\alpha_1\delta[n-2]+\ldots+\alpha_N\delta[n-N-1]$$

$$=x[n-1]$$

Similarly

$$Y(z)=z^{-n_0}X(z)$$

$$\Rightarrow y[n]=x[n-n_0]$$

3.3 A GENERAL Z-TRANSFORM FORMULA

We have seen that for a sequence $x[n]$ having support interval $0\le n\le N$ the z-transform is[5]

$$X(z)=\sum_{n=0}^{N}x[n]z^{-n} \qquad (7)$$

This definition extends for doubly infinite sequences having support interval $-\infty \leq n \leq \infty$ to

$$X(z) = \sum_{-\infty}^{\infty} x[n]z^{-n} \qquad (8)$$

3.4 REGION OF CONVERGENCE (ROC)

The Region of convergence (ROC) is the set of points z in the complex plane,
for which the summation is bounded (converges)[11][6]

$$\left|\sum_n x_n z^{-n}\right| < \infty$$

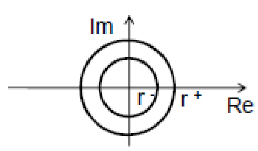
Since z is complex $z = re^{j\hat{w}}$

$$X(z) = \sum_n (x_n r^{-n})e^{-j\hat{\omega}n} = \sum_n \left|x_n r^{-n}\right| e^{-j\hat{\omega}n}$$

In general z –transform exist for

$$r^- < r > r^+$$

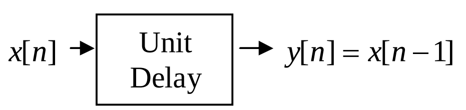
$$r^- < |z| < r^+$$

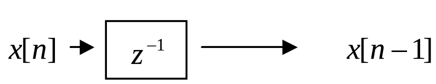


4. THE Z-TRANSFORM AS AN OPERATOR

The z-transform can be considered as an operator [7][8]

4.1 UNITS –DELAY OPERATOR





in the case of the unit delay, we observe that

$$y[n] = z^{-1}\{x[n]\} = x[n-1]$$

↑
Unit delay operator

Which is motivated by the fact that $Y(z) = z^{-1}X(z)$

Similarly , the filter

$$y[n] = x[n] - x[n-1]$$

Can be viewed as the operator

$$y[n] = (1 - z^{-1})\{x[n]\} = x[n] - x[n-1]$$

Since

$$Y(z) = X(z) - z^{-1}X(z) = (1 - z^{-1})X(z)$$

5. RELATION BETWEEN THE Z-DOMAIN AND THE FREQUENCY DOMAIN

$\hat{\omega}$ - Domain $\qquad\qquad$ z - Domain

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} \quad \textbf{Versus} \quad H(z) = \sum_{k=0}^{M} b_k z^{-k}$$

Comparing the above we see that the connection is setting $z = e^{j\hat{\omega}}$ in $H(z)$

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}} \qquad (9)$$

5.1 THE Z-PLANE AND THE UNIT CIRCLE

If we consider the z-plane, we see that $H(e^{j\hat{w}})$ corresponds to evaluating $H(z)$ on the unit circle[2]

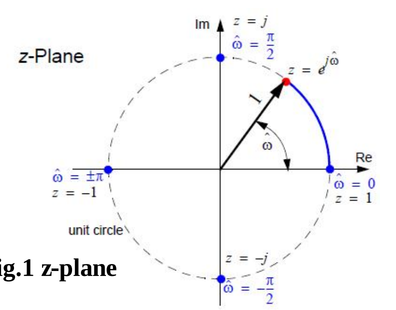


Fig.1 z-plane

From this interpretation we also can see why $H(e^{j\hat{\omega}})$ is periodic with period 2π

As $\hat{\omega}$ increase it continues to sweep around the unit circle over and over again [10]

5.2 GRAPHICAL RELATION BETWEEN z AND $\hat{\omega}$

when we make the substitution $z = e^{j\hat{w}}$ in $H(z)$ we know that we are evaluating the z -transform on the unit circle and thus obtain the frequency response. If we plot say $|H(z)|$ over the entire z-plane we can visualize how cutting out the response on just the unit circle, gives us the frequency response magnitude [10].

6. PROBLEM FORMULATION

The problem is to remove the noise in the x and y directions. To see this clearly , we use a clear image add noise to it and then try to remove the noise in the x and y direction using the Z-transform and a suitable digital filter. Z-transform converts a discrete-time signal into a complex frequency-domain representation. The complex variable z of the z-transform includes the used frequency where $z = e^{i\omega}$. Thus getting the z transform converts our problem from a problem in the time domain to a problem in the frequency domain choosing a specific value ω_1 in the frequency domain changes the problem to a problem in the ω domain at a certain frequency ω_1. Filtering , simply confines the frequency to a certain band $\omega_1 \leq \omega \leq \omega_2$

Fig.2 clear image

Fig.3 adding noise to a clear image

7. THE PROPOSED METHODOLOGY

On our software we do ,First we choose any picture then we convert this picture from image to data after convert we use it as points of x then we add noise to this point which we use after that to deploy the grid distance which use after that to find the frequencies from the equations. finally apply Z-transform which filtered the image shown below

D1 = Grid distance in x-direction = the width of image / used grid distance
Similarly;

D2 = Grid distance in y-direction = the length of image/ used grid distance

ω = 2*pi / grid distance in x and y direction

Where ω the frequency domain

8. Simulated Output Results

In this section we make use of the maximum measured frequencies in the x-direction ,y-direction which are connected to the minimum grid distance in the x and y direction changing the minimum grid distance used in the x and y direction changes the highest frequencies measured in the x and y direction.
These frequencies are the ones used in the deciding the lowest and highest frequencies chosen in the x and y directions respectively.
To illustrate the benefit of using different filters, we give results of filtering applied to certain image.

Used noisy image for testing by using the random different grid distance that given below then applying Z-transform .

D1=20
D2=40

Fig.4 used noisy image after applying Z-transform using the grid distance above

D1=15
D2=30

Fig.5 used noisy image after applying Z-transform using the grid distance above

D1=10
D2=20

Fig.6 used noisy image after applying Z-transform using the grid distance above

D1=5
D2=10

Of the photos displayed above, we find that the less distance increased frequency the best optimum frequencies we need to reduce the grid distance and thus we get the best pure image without any noise

CONCLUSION AND FUTURE WORK

From this output result we notice that the frequency is inversely proportional to grid Distance. It looks from the above work that changing the grid distances changes the frequency used. So we can choose a cut of frequency in either the x or the y direction to work with these cut off frequencies couid be changed and results couid be obtained subsequently.

In the future work we can using the frequency filters in the different form , possible using z-transform in 3D.

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