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Indexing Information Systems with Fuzzy Decision Attributes-An Approach through Rough Fuzzy Groups

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ABSTRACT

In 2005, G.Ganesan derived a procedure of adapting rough computing in fuzzy domains through thresholds. Through this, in 2008, he introduced a naive procedure of three way indexing any information system with fuzzy decision attributes. in 2013, he introduced a naïve procedure of characterizing an information system with fuzzy decision attributes through rough fuzzy groups. In this paper, we implement the indexing procedure on the characterization thus obtained.

***Keywords**: Rough Fuzzy Groups, Information System, indexing algorithm*

1. INTRODUCTION

Z.Pawalk's RS Model provides Two way approximations of a given concept under available knowledge. Since this theory finds a great impact in knowledge discovery, Dubois and Prade hybridized rough and fuzzy concepts and derived two concepts namely rough fuzziness and fuzzy roughness.

In 2005, G.Ganesan et. al., [2] introduced rough fuzzy groups. In that work, the closed fuzzy ordered pairs are defined for the elements of the given finite quotient group using rough fuzzy approximations. Also, independent to that work, in [3], G.Ganesan et. al., dealt the importance and the procedure of introducing a threshold in rough fuzzy approximation. In 2008, G.Ganesan et. al., [5] introduced an indexing algorithm in an information systems with fuzzy decision attributes. In 2013, G.Ganesan et. al., introduced a naïve algebraic procedure [4] of characterizing information system with fuzzy decision attributes. In this paper, we have combined the procedures described in [4] and [5].

This paper consists of five sections. In second section, we discuss the basic definitions, procedure of rough indexing and construction of rough fuzzy groups; In third section, we describe the basics of information systems; in fourth chapter, we describe the characterization and indexing procedures and the paper concludes with a few remarks as fifth section.

2. ROUGH SETS; ROUGH FUZZY SETS AND ROUGH FUZZY GROUPS

According to Z.Pawlak, for a given partition $W=\{X_1,X_2,...,X_t\}$ on a finite universe of discourse $U=\{x_1,x_2,...,x_n\}$, the lower and upper rough approximations of any subset A of U are given by $\underline{A}=\cup\{X_i\in W: X_i\subseteq A\}$ and $\overline{A}=\cup\{X_i\in W: X_i\cap A\neq\Phi\}$ respectively [6,7]. Later, Dubois and Prade introduced the concept of rough fuzzy sets [1], which describes the approximation of fuzzy subset of universe of discourse under the given partition. According to Dubois, For a given $U=\{x_1,x_2,...,x_n\}$, finite universe of discourse, $W=\{X_1,X_2,...,X_t\}$, an partition of U and a fuzzy subset A of U, firstly, we define $\mu_{\underline{A}}(X_i)=\inf\{\mu_A(x_j) : x_j\in X_i\}$ and $\mu_{\overline{A}}(X_i)=\sup\{\mu_A(x_j) : x_j\in X_i\}$. Through them, the lower and upper approximations of A are defined as $\underline{A}=(\mu_{\underline{A}}(X_1),\mu_{\underline{A}}(X_2),\ldots,\mu_{\underline{A}}(X_t))$ and $\overline{A}=(\mu_{\overline{A}}(X_1),\mu_{\overline{A}}(X_2),\ldots,\mu_{\overline{A}}(X_t))$ respectively. For each X_i, $(\mu_{\underline{A}}(X_i),\mu_{\overline{A}}(X_i))$ is called the fuzzy ordered pair [1].

2.1. Rough set approach on fuzzy sets using α and Rough Indices

Consider a set D, called **R-domain** [3], satisfying the following properties:

a) $D\subset(0,1)$
b) If a fuzzy set A is under computation, eliminate the values $\mu_A(x)$ and $\mu_{A^c}(x)$ $\forall x\in U$ from the domain D, if they exist.
c) After the computation using A, the values removed in (b) may be included in D provided A must not involve in further computation

Now, for any partition Let $\Psi=\{B_1,B_2,...,B_t\}$ on U, the given fuzzy set A, the lower and upper approximations with respect to α can be defined as $A_\alpha=\underline{(A[\alpha])}$ and $A^\alpha=\overline{(A[\alpha])}$ respectively where A[α] represents the strong α cut.

Now, we describe the lower indexing algorithm through RF Model in [5] as below:

It illustrates the method of indexing the elements of U, by using the lower and upper approximations of the given fuzzy set A [α is to be chosen from R-domain satisfying the property that α^{2n} and $\alpha^{n/2}$ are the members of R-Domain for any positive real number n]. Let M denote the largest number under consideration such that n+M is always positive and n-M is always negative for any integer n.

2.1.1. Algorithm

Algorithm index (x,A,α)
//Algorithm to obtain index of x an element of universe of discourse
//Algorithm returns the index

1. Let x_index be an integer initialized to 0
2. If $\mu_A(x)=1$
   begin

```
            x_index=M
            goto 6
      end
3.    If μ_A(x) = 0
      begin
            x_index=-M
            goto 6
      end
4.    compute A_α and A^α
5.    If x∈A_α
      while (x∈A_α)
        begin
            α=sqrt(α) //square root of α
            x_index=x_index+M+1
            compute A_α
        end
       else
            if x∉A^α
            while (x∉A^α)
              begin
           α=sqr(α) //square of α
           x_index=x_index-1-M

            compute A^α
         end
         else
            B=A; β=α
            compute B^β
            while (x∉A_α and x∈B^β)
             begin

      α=sqr(α) // square of α
      β=sqrt(β) // square root of β
            compute A_α,B^β

            x_index=x_index+1
             end
      if x∈A_α then x_index= - x_index
      end
6.    return x_index
```

By the above algorithm, `b' can be indexed by –1 and d can be indexed by –2-M. Similarly, other values of U can be indexed. These indices are called rough indices. Now, we describe the group structure on Rough Fuzziness as described in [2].

2.2. Rough Fuzzy Groups

Consider the congruence class Ω= $\left\{\vec{0},\vec{1},\ldots\ldots..,\overrightarrow{n-1}\right\}$ addition modulo n, where n is a positive integer. For any $\vec{p}$ *and* $\vec{q}$ in Ω, addition (modulo n) of $\vec{p}$ *and* $\vec{q}$ is defined by $\vec{p} +_n \vec{q}$ =(p+q)\n where (p+q)\n is the remainder obtained by dividing p+q by n.

Let F be any fuzzy subset of Z and every element of Ω be associated with a fuzzy ordered pair with respect to F through rough fuzzy approximations, say $\left(\left(\vec{0}_l,\vec{0}_u\right),\left(\vec{1}_l,\vec{1}_u\right),\ldots.,\left(\overrightarrow{(n-1)}_l,\overrightarrow{(n-1)}_u\right)\right)$. Now, the closeness of fuzzy ordered pairs are obtained as in [2] by using the following membership functions.

If the fuzzy ordered pair of $\vec{k}$ is $\left(\vec{k}_l,\vec{k}_u\right)$ under the fuzzy number F, define

$$\vec{k}_l^{(s+1)} = \max_{\vec{p}_l^{(s)} +_n \vec{q}_l^{(s)} = \vec{k}_l^{(s)}} \min(\vec{p}_l^{(s)},\vec{q}_l^{(s)})$$

and

$$\vec{k}_u^{(s+1)} = \max_{\vec{p}_u^{(s)} +_n \vec{q}_u^{(s)} = \vec{k}_u^{(s)}} \min(\vec{p}_u^{(s)},\vec{q}_u^{(s)})$$

where $\vec{p}_l^{\,0}=\vec{p}_l$; $\vec{p}_u^{\,0}=\vec{p}_u$ $\vec{q}_l^{\,0}=\vec{q}_l$; $\vec{q}_u^{\,0}=\vec{q}_u$; $\vec{k}_l^{\,0}=\vec{k}_l$ and $\vec{k}_u^{\,0}=\vec{k}_u$

Here, it may be observe that the above process is iterative and the iterative values converge or saturate or oscillate. As in [2], we call the saturated fuzzy ordered pair [or any one of the oscillated pairs] as the closed fuzzy ordered pair.

For example, Let $\Omega=\{\vec{0},\vec{1},\vec{2}\}$ and F any fuzzy subset of Z, which induce the fuzzy ordered pairs (0.3,0.7),(0.4,0.6) and (0.2,0.6) for $\vec{0}, \vec{1}$ *and* $\vec{2}$ respectively. Then $\vec{0}_l^{\,1}$ =max{min(0.3,0.3),min(0.2,0.4)}=0.3; $\vec{1}_l^{\,1}$ =max{min(0.4,0.3),min(0.2,0.2)}=0.3; $\vec{2}_l^{\,1}$ = max{min(0.3,0.2),min(0.4,0.4)} =0.4; $\vec{0}_u^{\,1}$ =max{min(0.7,0.7),min(0.6,0.6)}=0.7; $\vec{1}_u^{\,1}$ =max{min(0.7,0.6),min(0.6,0.6)} =0.6; $\vec{2}_u^{\,1}$ =max{min(0.7,0.6), min(0.6,0.6)}=0.6; Here the set of fuzzy ordered pairs in first iteration is $((0.3,0.7),(0.3,0.6),(0.4,0.6))$ and the second iteration also gives the set of fuzzy ordered pairs $((0.3,0.7),(0.3,0.6),(0.4,0.6))$.

This group of closed fuzzy ordered pairs is called ***rough fuzzy group on Ω***.

In [2] this concept is extended for any finite group (G,*) with cardinality m and $\Omega=\{\vec{0},\vec{1},\ldots\ldots,\overline{\vec{k}-1}\}$, through the following algorithm:

a) begin
b) Consider a given finite group G; a congruence group of Z under addition modulo k and an epimorphism f :G→Ω.
c) Input ((G,*),Ω,f, F)
d) Compute the quotient group L=G/K
e) By the axiom of choice, denote each element of L by any of it member.
f) (For example, if $a\in K_x$ for some $x\in G$, then denote K_x by [a])
g) Compute closed rough fuzzy ordered pair for each element of Ω using the fuzzy subset F of Z.
h) Associate the closed rough fuzzy ordered pair with the preimage of each element of Ω in G/K.
i) The group L associated with the fuzzy ordered pairs is called the rough fuzzy group, denoted by (G,*,Ω,f,F).
j) return

This algorithm is illustrated by using the following example.

Example 2.2.1*:* Consider a group G={1,ω,ω²,ω³, ω⁴,ω⁵} where ω is the sixth root of unity, with the binary operation*, which is defined as below:

*	1	ω	ω^2	ω^3	ω^4	ω^5
1	1	ω	ω^2	ω^3	ω^4	ω^5
ω	ω	ω^2	ω^3	ω^4	ω^5	1
ω^2	ω^2	ω^3	ω^4	ω^5	1	ω
ω^3	ω^3	ω^4	ω^5	1	ω	ω^2
ω^4	ω^4	ω^5	1	ω	ω^2	ω^3
ω^5	ω^5	1	ω	ω^2	ω^3	ω^4

Let f be any epimorphism defined from G onto $\Omega=\left\{\vec{0},\vec{1},\vec{2}\right\}$ with kernel K and let G/K={{1, ω^3},{ω^2, ω^5}, {ω,ω^4}}.

By axiom of choice, denote the sets {1, ω^3},{ω^2, ω^5} and {ω,ω^4} by [1],[ω^2] and [ω] respectively. Let the isomorphism φ:G/K $\rightarrow\Omega$ be given by $\varphi([1])=\vec{0}$; $\varphi([\omega])=\vec{2}$ and $\varphi([\omega^2])=\vec{1}$. Let F be any fuzzy subset of Z, which give the approximations for F be (0.3,0.4,0.2) and (0.7,0.6,0.6) respectively. Then the set of closed fuzzy pairs of Ω is given by ((0.3,0.7),(0.4,0.6),(0.2,0.6)).

Now, by considering the preimage of φ, for each element of G/K, the closed ordered pair can be associated. Thus, the rough fuzzy group can be obtained.

Here, one may observe that the number of Rough Fuzzy Groups on G/K is equal to the number of epimorphisms defined from G/K to Ω.

Now, in the next section, we implement these concepts in the information systems.

3. INFORMATION SYSTEMS

Consider the decision table of any information system [7] T=(U, A, C, D), where U is the set of all records, A is a set of primitive features, C and D are the subsets of A namely the conditional and decision features respectively. Let IND (P)={(x,y)$\in$UxU : a(x)=a(y) for all a in P} where P is a subset of A. Obviously, IND(P) partitions U into disjoint blocks with respect to P and this collection is denoted with U/IND (P) or U/P. For the indiscernibility relation IND(P), the lower and upper approximations are defined as $\underline{P}X=\cup\{Y\in U/P:Y\subseteq X\}$ and $\overline{P}X=\cup\{Y\in U/P:Y\cap X\neq\Phi\}$ respectively. The classes U/IND(C) and U/IND (D) are called condition and decision classes respectively and the C-Positive region of D is given by $POS_C(D)=\bigcup_{X\in U/D}\underline{C}X$.

For any c$\in$C, if $POS_{C-\{c\}}(D)\neq POS_C(D)$ then c is said to be indispensable in T and hence deleting it from T makes T to be inconsistent. T is said to be independent if all the features of it are indispensable. A set of features R in C is called a reduct or minimal feature, if T'=(U, A, R, D) is independent and $POS_R(D)=POS_C(D)$. The set of all features indispensable in C is denoted by CORE(C) [7]. In other words, CORE(C) = $\cap$RED(C) where RED(C) is the set of all reducts of C. The elementary method of computing reduct was introduced by Skowron through discernibility matrices. At present, several methods are in use and among them one of the popular one is Quick Reduct Algorithm which is given below:.

3.1. QUICKREDUCT(C,D)

C, the set of all conditional attributes;
D, the set of decision attributes.
(a) R $\leftarrow$ {}
(b) Do
(c) T $\leftarrow$ R
(d) x $\in$ (C-R)
(e) if $g_{R\cup\{x\}}(D) > g_T(D)$, where
$g_R(D)$=card($POS_R(D)$) / card(U)

(f) $T \leftarrow R \cup \{x\}$
(g) $R \leftarrow T$
(h) until $g_R(D) == g_C(D)$
(i) return R

4. INDEXING INFORMATION SYSTEMS WITH FUZZY DECISION ATTRIBUTES

In this section, first we characterize the given information system with fuzzy decision attributes as discussed in [4] and we index it as given in [5].

4.1 Characterization in Information System

Consider any information system with fuzzy decision attributes. As in [4], we characterize it through the following algorithm:

4.1.1 Algorithm

a) Begin
b) Using a threshold chosen from D, apply Quick Reduct Algorithm and Compute the possible Reducts say $A_1, A_2, \ldots, A_k$.
c) For each reduct A_j, cluster the records of the information system.
d) Define the fuzzy ordered pair for each of the cluster obtained by using Rough Fuzzy Approximations.
e) Suppose that the number of clusters obtained using A_j be n_j, consider a group Z_{nj} of integers Congruence modulo n_j.
f) Now, it is observed that there are $n_j!$ ways to define bijections between the clusters and the group Z_{nj}.
g) For each bijection, we obtain a Rough Fuzzy Group of Clusters of the records of the Information System.
h) End

This algorithm is illustrated with the follwoing example.
Consider any decision table given by

	a	b	c	d	μ_E	$E[\alpha]$
x_1	1	0	2	1	0.7	1
x_2	1	0	2	0	0.8	1
x_3	1	2	0	0	0.3	0
x_4	1	2	2	1	0.4	0
x_5	2	1	0	0	0.9	1
x_6	2	1	1	0	0.2	0
x_7	2	1	2	1	0.6	1

where a,b,c,d are the conditional attributes and E is the fuzzy decision attribute with α=0.5 . On applying Quick Reduct algorithm, the Reducts {b,c} and {b,d} are obtained.

Now, through the reduct {b,c}, the indiscernibility classes obtained are $\{\{x_1,x_2\},\{x_3\},\{x_4\},\{x_5\},\{x_6\},\{x_7\}\}$ and hence the corresponding mapping of fuzzy ordered pairs are

$$\{x_1,x_2\} \leftarrow (0.7,0.8)$$
$$\{x_3\} \leftarrow (0.3, 0.3)$$
$$\{x_4\} \leftarrow (0.4,0.4)$$
$$\{x_5\} \leftarrow (0.9,0.9)$$
$$\{x_6\} \leftarrow (0.2,0.2)$$
$$\{x_7\} \leftarrow (0.6,0.6)$$

On applying the following bijection with the indiscernibility classes and $\Omega = \left\{\vec{0},\vec{1},\ldots\ldots,\vec{5}\right\}$, the congruence group of Z under addition modulo 6, we have

$\{x_1,x_2\} \leftarrow \bar{1} \leftarrow (0.7,0.8)$
$\{x_3\} \leftarrow \bar{4} \leftarrow (0.3, 0.3)$
$\{x_4\} \leftarrow \bar{5} \leftarrow (0.4,0.4)$
$\{x_5\} \leftarrow \bar{2} \leftarrow (0.9,0.9)$
$\{x_6\} \leftarrow \bar{3} \leftarrow (0.2,0.2)$
$\{x_7\} \leftarrow \bar{0} \leftarrow (0.6,0.6)$

In this case, set of two closed fuzzy ordered pairs are obtained for $\Omega= \{\vec{0},\vec{1},........,\vec{5}\}$ as follows:

$\bar{0} \leftarrow (0.7,0.8)$	$\bar{0} \leftarrow (0.7,0.8)$
$\bar{1} \leftarrow (0.7,0.8)$	$\bar{1} \leftarrow (0.7,0.8)$
$\bar{2} \leftarrow (0.7,0.8)$	$\bar{2} \leftarrow (0.9,0.9)$
$\bar{3} \leftarrow (0.7,0.8)$	$\bar{3} \leftarrow (0.7,0.8)$
$\bar{4} \leftarrow (0.9, 0.9)$	$\bar{4} \leftarrow (0.7, 0.8)$
$\bar{5} \leftarrow (0.7,0.8)$	$\bar{5} \leftarrow (0.7,0.8)$

Hence, for α=0.5 characterization with respect to the reduct {b,c} are given by

$\{x_1,x_2\} \leftarrow \bar{1} \leftarrow (0.7,0.8) \leftarrow (0.7,0.8)$
$\{x_3\} \leftarrow \bar{4} \leftarrow (0.3, 0.3) \leftarrow (0.9, 0.9)$
$\{x_4\} \leftarrow \bar{5} \leftarrow (0.4,0.4) \leftarrow (0.7,0.8)$
$\{x_5\} \leftarrow \bar{2} \leftarrow (0.9,0.9) \leftarrow (0.7,0.8)$
$\{x_6\} \leftarrow \bar{3} \leftarrow (0.2,0.2) \leftarrow (0.7,0.8)$
$\{x_7\} \leftarrow \bar{0} \leftarrow (0.6,0.6) \leftarrow (0.7,0.8)$

and

$\{x_1,x_2\} \leftarrow \bar{1} \leftarrow (0.7,0.8) \leftarrow (0.7,0.8)$
$\{x_3\} \leftarrow \bar{4} \leftarrow (0.3, 0.3) \leftarrow (0.7, 0.8)$
$\{x_4\} \leftarrow \bar{5} \leftarrow (0.4,0.4) \leftarrow (0.7,0.8)$
$\{x_5\} \leftarrow \bar{2} \leftarrow (0.9,0.9) \leftarrow (0.9,0.9)$
$\{x_6\} \leftarrow \bar{3} \leftarrow (0.2,0.2) \leftarrow (0.7,0.8)$
$\{x_7\} \leftarrow \bar{0} \leftarrow (0.6,0.6) \leftarrow (0.7,0.8)$

4.2 Indexing Procedure

Now consider the information system characterization with α=0.5 as below:

$\{x_1,x_2\} \leftarrow (0.7,0.8)$
$\{x_3\} \leftarrow (0.7, 0.8)$
$\{x_4\} \leftarrow (0.7,0.8)$
$\{x_5\} \leftarrow (0.9,0.9)$
$\{x_6\} \leftarrow (0.7,0.8)$
$\{x_7\} \leftarrow (0.7,0.8)$

Here, $K=\{\{x_1,x_2\},\{x_3\},\{x_4\},\{x_5\},\{x_6\},\{x_7\}\}$ for the reduct (b,c} and μ_E=(0.7, 0.8,0.3,0.4,0.9,0.2,0.6). Hence $E[\alpha]=\{x_1,x_2,x_5,x_7\}$. therefore, by applying the indexing algorithm, we obtain the indices on the characterization of information system as follows:

$\{x_1,x_2\} \leftarrow$ M+1
$\{x_3\} \leftarrow$ -1-M
$\{x_4\} \leftarrow$ -1-M
$\{x_5\} \leftarrow$ M+2
$\{x_6\} \leftarrow$ -2-M
$\{x_7\} \leftarrow$ M+1

5. CONCLUSION

In this paper, we have discussed the concept of indexing the information system which is algebraically characterized through rough fuzziness.

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