ISSN 2347 - 3983

International Journal of Emerging Trends in Engineering Research (IJETER), Vol. 3 No.1, Pages : 01 – 04 (2015)

Special Issue of ICEEMC 2015 - Held during January 27, 2015, Chennai, India

Generalization of Some Fibonacci Data Models

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Abstract— In this paper, we investigate some data models of Fibonacci codes. Three-dimensional vector model, multinomial vector model, quaternion and octet data models were presented. It is shown that real numbers can be represented as integers. Similarly, complex numbers are also denoted by a collection of corresponding integers. These Fibonacci models increase the accuracy of computer data representation.

Keywords*:* Fibonacci numbers, data models, three-dimensional vector model, multinomial vector model, quaternion, octet data model.

INTRODUCTION

The rapid increase in Information Technology leads to the development of new types of arithmetic applications to be used in well-developed digital devices [1]. Enhancement of the performance of these digital devices and information processing has resulted in the development of arithmetic codes. For example, in efficient data transmission, different codes such as Hamming codes, cyclic codes and different weight codes were used [2-5]. To build up sub-systems, there are different varieties of codes that can be used. Consequently, the efficiency of systems will decrease as a result of the requirement of converters to link these subsystems. These codes use the binary numbering system, which leads to several disadvantages with respect to performance of these devices. This will, in turn affect the accuracy and precision. Currently, no general purpose code is available that can be used to perform arithmetic operations, transmission, and information processing [8-10]. In this paper different data models for Fibonacci codes will be represented in order to make Fibonacci codes the base for general purpose codes.

THREE-DIMENSIONAL VECTOR MODEL

The presentation of the three-dimensional vector $\bar{\mathbf{a}}$ =xi+yj+zk, where x, y and z - are integer numbers, are provided by base, whose elements are calculated using the following from:

$$w_{l+3} = w_{l+2} + w_l \quad \text{for } l = 0, 1, 2,...;$$

$$w_l = w_{l+3} - w_{l+2} \quad \text{for } l = -1, -2,...,$$

where w0 = i, w1 = j, w2 = k.

Directly i element can be described by using the formula below:

$$wl = \varphi_2(l-2)i + \varphi_2(l-3)j + \varphi_2(l-1)k. \qquad (1)$$

The initial values of il have the following sequence:

l	7	6	5	4	3	2	1	0
wl	3i+2j+4k	2i+j+3k	i+j+2k	i+j+k	i+k	k	j	i

l	-1	-2	-3	-4	-5	-6	-7
wl	-j+k	-i+j	i+j-k	i-2j+k	-2j+k	3j-2k	3i-2j

The Mathematical objects $\psi_3(l)$ described by formula (1), can be called the Fibonacci three-dimensional vector.

By using the sequence $\mathbf{\Psi}_3 = \{\psi_3(l)\}$, the three-dimensional vectors can also be described in the following form:

$$xi+yj+zk = \sum_{l=-m}^{n} a_l \psi_3(l) \qquad (2)$$

where $a_l \in \{-1;0;1\}$

This Sequence, $\mathbf{\Psi}_3$,may be formed by adding three sequences:

$$\mathbf{\Psi}_3 = \sigma^2(\varphi_2^*)_{i+}\sigma^3(\varphi_2^*)_{j+}\sigma(\varphi_2^*)_{k.}$$

From the above and, in order to obtain a 2-code Fibonacci three-dimensional vector xi+yj+zk it is necessary to present x in the base of $\sigma^2(\varphi_2^*)$, y - in the base of $\sigma^3(\varphi_2^*)$, but z – in the base of $\sigma(\varphi_2^*)$ and adding their codes.

As previously shown; the Fibonacci three-dimensional vector model can be expressed as follows:

$$\mathbf{\Phi}_{\bar{\mathbf{a}}} = \langle \mathbf{K}_2, \mathbf{A}; \varphi_{\bar{\mathbf{a}}}, P(a) \rangle,$$

where $\mathbf{K}_2 = \{a_{jn}a_{j(n-1)}...a_{j1}a_{j0}...a_{j(-m)}\}$ - is a set representing an extended Fibonacci 2-codes;

$\mathbf{A} = \{-1;0;1\}$- automate alphabet;

$\varphi_{\bar{\mathbf{a}}}$ - the unique attitude "to correspond to the expression (2) for $a_l \in \mathbf{A}$ ";

$P(a)$- the unique attitude " corresponding to the set $\{a_l\}$ in accordance to the previously mentioned characteristics ".

MULTINOMIAL FIBONACCI MODEL

An approximate solution for the integral function F(x) usually change, on one multinomial length of [a,b], or breaking down length [a,b] into several parts, and for each part an algebraic multinomial first, second, or third- degree will be selected and carried out.

In this work, these two approaches representing the function in the form of multinomial will be chosen.

For realization of the first approach one polynomial G(x) of r degree is used:

$$F(x) \approx G(x) = \sum_{i=0}^{r} k_i x^i \ .$$

where $x^0, x^1, \ldots, x^r$ can form the base of the linear space that have the dimension r+1. Hence it follows the equivalence of the multinomial G(x) to point (r+1)-dimensional space. So multinomial G(x) with circular factor can be presented in base, whose elements are (p+1)-dimension of Fibonacci mathematical objects for r=p.

The Second approach uses the multinomial group of third degree, each of which can replace the function F(x) with the corresponding part of [a,b].

It can be said that $F(x) = \{g_i(x)\}$,

Where $g_i(x) = k_{3i}x^3 + k_{2i}x^2 + k_{1i}x \ + k_{0i}$ (3)

However, G(x) can be presented as multinomial combination of third degree of the type shown in (3)

$$G(x) = \sum_{i=0}^{k} x^{4i} g_i(x) \ ,$$

where $k = \left]\frac{r-3}{4}\right[$.

It is apparent from the above, that two approaches leads to the same problematic situation that is the presentation of multinomial third degree. Hence forth, the presentation of such multinomial will be expressed here in the form:

$$g(x) = k_3x^3 + k_2x^2 + k_1x \ + k_0 = \sum_{l=-m}^{n} a_l w_l \qquad (4)$$

In this case elements of the base will be calculated using the following from:

wl+4 = wl+3 + wl for l = 0, 1, 2,...;
wl = wl+4 - wl+3 for l = - 1, -2,...,
where w0=1, w1= x, w2= x2, w3= x3 .

The direct form is determining w_l will be:

$$w_l = \varphi_3(l-2)x^3 + \varphi_3(l-5)x^2 + \varphi_3(l-4)x + \varphi_3(l-3) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The Mathematical objects $\psi_4^{\Pi}(l)$ described by formula (5), called the Fibonacci multinomial of third degree. And the sequence $\Psi_4^{\Pi} = \{\psi_4^{\Pi}(l)\}$ Fibonacci multinomial third degree can be obtained by adding the four sequences:

$$\Psi_4^{\Pi} = \sigma^3(\varphi_3^*) + \sigma^4(\varphi_3^*)x + \sigma^5(\varphi_3^*)x^2 + \sigma^2(\varphi_3^*)x^3 \ .$$

Presentation of k_0 in the base of $\sigma^3(\varphi_3^*)$, - k_1 in the base of $\sigma^4(\varphi_3^*)$, k_2, - in the base of $\sigma^5(\varphi_3^*)$ and k_3 - in the base of $\sigma^2(\varphi_3^*)$ the addition of their codes, of 3-code Fibonacci multinomial of third degree will be obtained.

From the mathematical analysis previously discussed: the following model of the multinomial of third degree were concluded or reached:.

Taking into consideration the ideas that have been discussed earlier, we come to the following model of the multinomial of third degree:

$$\mathbf{\Phi}_G = \langle \mathbf{K}_3, \mathbf{A}; \varphi_G, P(a) \rangle,$$

where $\mathbf{K}_3 = \{a_{jn} a_{j(n-1)} \ldots a_{j1} a_{j0} \ldots a_{j(-m)}\}$ - is the Fibonacci 3-code sequence;

$\mathbf{A} = \{-1;0;1\}$ - - automate alphabet;

φ_G - is the unique attitude "to correspond to the expression (22) for $a_l \in \mathbf{A}$ ";

$P(a)$ - a the unique attitude "to correspond to the expression $\{a_l\}$ with according to the previously mentioned characteristics ".

QUATERNION MODEL

For computer presentation quaternion type H=a+bi+cj+dk, where a, b, c and d - integer numbers, are suggested to use the base, which elements can be calculated in the formula:

wl+4= wl+3+ wl , for l = 0, 1, 2,...;
wl = wl+4- wl+3 , for l = - 1, -2,...,
where w0 = 1 , w1 = i , w2 = j ,w3 = k.

The direct form for calculation wl is:

$$w_l = \varphi_3(l-3) + \varphi_3(l-4)i + \varphi_3(l-5)j + \varphi_3(l-2)k \quad (6)$$

For initial values l we have the sequence :

l	11	10	9	8
w_l	$(5+4i+3j+7k)$	$(4+3i+2j+5k)$	$(3+2i+j+4k)$	$(2+i+j+3k)$

l	7	6	5	4	3	2	1	0
w_l	$(1+i+j+2k)$	$(1+i+j+k)$	$(1+i+k)$	$(1+k)$	k	j	i	1

l	-1	-2	-3	-4	-5
w_l	$(-j+k)$	$(-i+j)$	$(-1+i)$	$(1+j-k)$	$(i-2j+k)$

l	-6	-7	-8	-9
w_l	$(1-2i+j)$	$(-2+i-j+k)$	$(1-i+3j-2k)$	$(-1+3i-3j+k)$.

The Mathematical objects $\psi_4^{\kappa}(l)$ described by formula (6), are called Fibonacci Quaternion.

Using series $\mathbf{\Psi}_4^{\kappa}$ ={ $\psi_4^{\kappa}(l)$ } and alphabet $\mathbf{A} = \{-1;0;1\}$, it is possible to represent quaternion in the form:

$$a+bi+cj+dk = \sum_{l=-m}^{n} a_l \psi_4^{\kappa}(l), \quad (7)$$

The series of $\mathbf{\Psi}_4^{\kappa}$ can be formed by summation of four sequences:

$$\mathbf{\Psi}_4^{\kappa} = \sigma^3(\varphi_3^*) + \sigma^4(\varphi_3^*)i + \sigma^5(\varphi_3^*)j + \sigma^2(\varphi_3^*)k.$$

The quaternion code a+bi+cj+dk is obtained by adding a ones of the type 1, b ones of the type i, c ones of the type j and d ones of the type k. The sign of the ones is defined by the corresponding sign to each numbers.

Presenting a in base $\sigma^3(\varphi_3^*)$, b - in base $\sigma^4(\varphi_3^*)$, c - in base $\sigma^5(\varphi_3^*)$, and d - in base $\sigma^2(\varphi_3^*)$ and summation their codes, we obtain 3-code Fibonacci quaternion.

With the respect to the above said, we have the following Fibonacci quaternion model:

$$\mathbf{\Phi}_H = \langle \mathbf{K}_3, \mathbf{A}; \varphi_H, P(a) \rangle,$$

where $\mathbf{K}_3 = \{a_{jn} a_{j(n-1)} \ldots a_{j1} a_{j0} \ldots a_{j(-m)}\}$ an extended set 3- Fibonacci codes;

$\mathbf{A} = \{-1;0;1\}$- automate alphabet;

φ_H - the unique attitude "to correspond to the expression (2) for $a_l \in \mathbf{A}$ ";

$P(a)$- the unique attitude "to correspond to the set $\{a_l\}$ with according to the previously mentioned characteristics ".

Octet Model

The octet presentation

$$O = \gamma_1 + \gamma_2 i + \gamma_3 j + \gamma_4 k + h_1 e + h_2 ie + h_3 je + h_4 ke,$$

where $\gamma_1, \gamma_2, \gamma_3, \gamma_4, h_1, h_2, h_3$ i h_4 - integer numbers , of type

$$O = \sum_{l=-m}^{n} a_l w_l \ , \ a_l \in \{-1;0;1\} \qquad (8)$$

Use the base, which elements are calculated by formula:

$$w_{l+8} = w_{l+7} + w_l, \ \text{for } l = 0, 1, 2,...;$$
$$w_l = w_{l+8} - w_{l+7}, \ \text{for } l = -1, -2,...,$$

where $w_0 = 1$, $w_1 = i$, $w_2 = j$, $w_3 = k$, $w_4 = e$, $w_5 = ie$, $w_6 = je$, $w_7 = ke$.

The direct form for determination wl has the following form:

$$w_l = \varphi_7(l-7) + \varphi_7(l-8)i + \varphi_7(l-9)j + \varphi_7(l-10)k +$$
$$+ \varphi_7(l-11)e$$
$$+ \varphi_7(l-12)ie + \varphi_7(l-13)je + \varphi_7(l-6)ke \quad . \qquad (9)$$

The Mathematical objects $\psi_8(l)$ described by formula (10), are called the Fibonacci octave.

The sequence $\mathbf{\Psi}_8 = \{ \psi_8(l) \}$ Fibonacci octet can be formed as summation of eight sequences:

$$\mathbf{\Psi}_8 = \sigma^7(\varphi_7^*) + \sigma^8(\varphi_7^*)i + \sigma^9(\varphi_7^*)j + \sigma^{10}(\varphi_7^*)k + \sigma^{11}(\varphi_7^*)e +$$
$$+ \sigma^{12}(\varphi_7^*)ie + \sigma^{13}(\varphi_7^*)je + \sigma^6(\varphi_7^*)ke.$$

In order to obtain the code of the octave, it is necessary to get the summation of the numbers codes $\gamma_1, \gamma_2, \gamma_3, \gamma_4$, h_1, h_2, h_3 and h_4, which are presented in bases of $\sigma^7(\varphi_7^*), \sigma^8(\varphi_7^*), \sigma^9(\varphi_7^*), \sigma^{10}(\varphi_7^*), \sigma^{11}(\varphi_7^*), \sigma^{12}(\varphi_7^*), \sigma^{13}(\varphi_7^*)$ and $\sigma^6(\varphi_7^*)$, respectively.

Coming from the above mentioned, we have the following Fibonacci octave model:

$$\mathbf{\Phi}_O = \langle \mathbf{K}_7, \mathbf{A}; \varphi_O, P(a) \rangle$$

where $\mathbf{K}_7 = \{a_{jn} a_{j(n-1)} ... a_{j1} a_{j0} ... a_{j(-m)}\}$ -the extended set of Fibonacci 7-codes;

$\mathbf{A} = \{-1;0;1\}$ - automate alphabet;

φ_O -a unique attitude "to correspond to the expression (7) for $a_l \in \mathbf{A}$ ";

$P(a)$- a unique attitude "to correspond to the set $\{a_l\}$ with according to the previously mentioned

CONCLUSION

For reduction of inaccuracy of the computer presentation of data and removal of inaccuracy of the calculations it is reasonable to generalize data models for Fibonacci codes. In this work some Fibonacci data models were represented in order to use them in digital systems.

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