

Statistical Properties of Amplify and Forward Relay Fading Channels



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Abstract: Cooperation diversity schemes for cellular networks permit mobile stations to relay signals to a final destination, thereby increasing the network capacity and coverage. The mobile relays either decode and retransmit the received signal or simply amplify and forward (A & F) the signal. In this paper we study and simulate the statistical properties of amplify and forward relay (A & F) fading channels such as, autocorrelation, doppler spectrum and level crossing rate.

Keywords: Cooperation diversity, Rayleigh fading, Relay channels.

1. Introduction

In telecommunications, a diversity scheme refers to a method for improving the reliability of a message signal by using two or more communication channels with different characteristics. Diversity plays an important role in combatting fading and co-channel interference and avoiding error bursts. It is based on the fact that individual channels experience different levels of fading and interference. Multiple versions of the same signal may be transmitted and/or received and combined in the receiver. Alternatively, a redundant forward error correction code may be added and different parts of the message transmitted over different channels. Diversity techniques may exploit the multipath propagation, resulting in a diversity gain, often measured in decibels.

Cooperative diversity is a cooperative multiple antenna technique for improving or maximising total network channel capacities for any given set of bandwidths which exploits user diversity by decoding the combined signal of the relayed signal and the direct signal in wireless multihop networks.

In a cooperation diversity scheme, one MS partners with another MS to send (or receive) its signal to (from) the base station (BS) or some

other final destination. The partner station serves as a relay, forwarding the signal from the source to the destination. This provides receive or transmit antenna diversity (depending on the link—downlink or uplink) in a virtual fashion, since multiple transmit or receive antennas are used although they are not colocated. Such systems, also termed virtual antenna array (VAA) systems, promise to increase the system capacity and coverage.

Cooperation diversity systems can be broadly categorized as either non-regenerative or regenerative depending on the relay functionality. In the former, the relay simply amplifies and forwards (A & F) the received signal, while in the latter the relay decodes, encodes, and forwards the received signal. The A & F mode puts less processing burden on the relay and, hence, is often preferable when complexity and/or latency issues are important. This paper primarily focuses on A & F systems.

2. Relay Fading Channel Model

Consider the downlink relay fading channel from a (stationary) BS to a destination MS via a mobile relay (R), as shown in Fig.1.

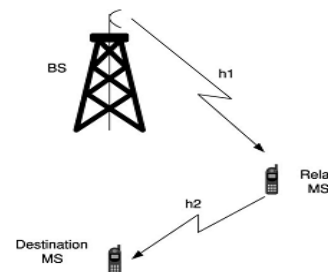


Fig.1: Relay fading channel scenario.

Assuming flat fading, the signal received by the relay at time t is

$$r_1(t) = h_1(t)s(t) + n_1(t)$$

where,

$r_1(t)$, is signal received by relay at time t .

$h_1(t)$, is the channel between the BS and relay with power σ_1^2 .

$s(t)$, is the transmitted signal with average power E_1 .

$n_1(t)$, is the (AWGN), with power σ_n^2 .

The relay amplifies $r_1(t)$ and retransmits it to the destination MS, which receives it as,

$$r_2(t) = A(t)h_2(t)r_1(t) + n_2(t)$$

where,

$A(t)$, is the amplification factor scaling the power transmitted by relay.

$h_2(t)$, is the channel between relay and destination MS, with power σ_2^2 .

$n_2(t)$, is zero mean AWGN with power σ_n^2 .

2.1. Amplification Factor A

For a fixed gain relay channel [1],

$$A_1 = \sqrt{\frac{E_2}{\mathbf{E}[|r_1|^2]}} = \sqrt{\frac{E_2}{E_1\sigma_1^2 + \sigma_n^2}}$$

For a variable gain relay channel [2].

$$A_2 = \sqrt{\frac{E_2}{E_2|h_1|^2 + \sigma_n^2}}$$

3. Time-Domain Correlations

Most of our results continue under the assumption of isotropic antennas operating in a two-dimensional (2-D) isotropic scattering environment, corresponding to non-line-of-sight (NLOS) conditions at the relay station, an assumption justified by the low elevation of the relay and destination terminals. The local area around the BS is assumed to be scatterer free owing to its high elevation. The auto-correlations of $x(t)$, $y(t)$, and $h(t)$ are [3],

$$R_{xx}(\tau) = R_{yy}(\tau) = \mathbf{E}[x(t+\tau)x(t)] \\ = \frac{\sigma_1^2\sigma_2^2}{2} J_0(2\pi f_1\tau) J_0(2\pi \hat{f}_1\tau) J_0(2\pi f_2\tau)$$

$$R_{hh}(\tau) = \frac{1}{2} \mathbf{E}[h(t+\tau)h^*(t)] \\ = \frac{\sigma_1^2\sigma_2^2}{2} J_0(2\pi f_1\tau) J_0(2\pi \hat{f}_1\tau) J_0(2\pi f_2\tau)$$

$$f_1 = \frac{v_1}{\lambda_1}, \quad \hat{f}_1 = \frac{v_1}{\lambda_2}, \quad f_2 = \frac{v_2}{\lambda_2}$$

where $J_0(x)$ is the zeroth order Bessel function of the first kind; f_1 is the maximum Doppler shift induced by the motion of the relay with speed v_1 in the BS-relay link having a carrier wavelength of λ_1 ; and \hat{f}_1 and f_2 are the maximum Doppler shift induced by the motion of the relay and the destination MS, respectively, in the relay-destination MS link (carrier wavelength of λ_2). The auto-correlation can be derived as [4],

$$R_{hh}(\tau) = \frac{1}{2} \mathbf{E}[h_1(t+\tau)h_2(t+\tau)h_1^*(t)h_2^*(t)] \\ = \frac{1}{2} \mathbf{E}[h_1(t+\tau)h_1^*(t)] \mathbf{E}[h_2(t+\tau)h_2^*(t)] \\ = 2R_{h_1h_1}(\tau)R_{h_2h_2}(\tau) \\ = 2 \left\{ \frac{\sigma_1^2}{2} J_0(2\pi f_1\tau) \right\} \left\{ \frac{\sigma_2^2}{2} J_0(2\pi \hat{f}_1\tau) J_0(2\pi f_2\tau) \right\} \\ = \frac{\sigma_1^2\sigma_2^2}{2} J_0(2\pi f_1\tau) J_0(2\pi \hat{f}_1\tau) J_0(2\pi f_2\tau) \quad (4)$$

Simplified autocorrelation function is given by,

$$R_{hh}(\tau) = \frac{\sigma_1^2\sigma_2^2}{2} J_0(2\pi f_1\tau)^2 J_0(2\pi f_2\tau) \\ = \frac{\sigma_1^2\sigma_2^2}{2} J_0(2\pi f_1\tau)^2 J_0(2\pi a f_1\tau)$$

where $a = f_2/f_1 = v_2/v_1$ is the ratio of two Doppler shifts.

4. Doppler Spectrum and Doppler Spread

The Doppler power spectral density of fading channel describes how much spectral broadening it causes. The Doppler spectra are a function of maximum Doppler frequency f_1 and the Doppler ratio a . The maximum Doppler frequency in relay channels is $f_{\max} = f_1 + \hat{f}_1 + f_2$.

Doppler Spread of a channel h with autocorrelation function $R_{hh}(T)$ is given by [5],

$$B_d = \frac{1}{2\pi} \sqrt{\left(\frac{\dot{R}_{hh}(\tau)}{R_{hh}(0)} \right)^2 - \frac{\ddot{R}_{hh}(\tau)}{R_{hh}(0)}}$$

Using known auto-correlation functions for various channels we get the Doppler spread as

$$\text{Cellular channel: } B_d = \sqrt{\frac{f_1^2}{2}}$$

$$\text{Mobile-to-Mobile channel: } B_d = \sqrt{\frac{f_1^2 + f_2^2}{2}}$$

$$\text{Relay channel: } B_d = \sqrt{f_1^2 + \frac{f_2^2}{2}}$$

The relay fading channels have a higher Doppler spread when compared to cellular channels and mobile-to-mobile channels. For example, with $f_1 = f_2$, the Doppler spread for relay channels is approximately 70% and 25% higher than cellular channels and mobile-to-mobile channels, respectively (assuming f_1 is the same in all cases).

6. Level Crossing Rate (LCR)

The LCR is defined as the rate at which the channel envelope crosses a specified level with a positive slope. The LCR governs the average fade duration (AFD) and affects handoff and outage probability statistics at the system level. The LCR of the envelope α at level R for the relay fading channel h is given by [6],

$$L_\alpha(\alpha = R) = \frac{4\sqrt{\pi}R}{\sqrt{2}\sigma_1^2\sigma_2^2} \int_0^\infty \frac{1}{y^2} \exp\left(-\frac{\sigma_2^2 R^2 + \sigma_1^2 y^4}{\sigma_1^2 \sigma_2^2 y^2}\right) \cdot \sqrt{\sigma_1^2 f_1^2 y^4 + \sigma_2^2 (f_1^2 + f_2^2) R^2} dy.$$

As α increases, the LCR increases due to increased mobility in the channel. The LCRs of relay channel are higher than cellular channels for a given envelope level R.

7. Results

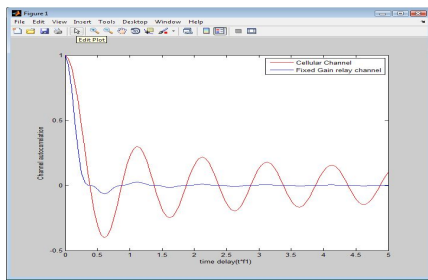


Fig.2: Channel Autocorrelation vs Normalized time delay.

Fixed gain channel is found to have rapid decorrelation which is useful when employing diversity to combat fading.

Doppler Spectrum

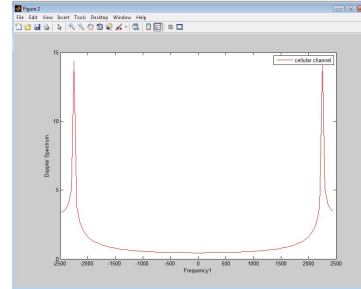


Fig.3: Doppler Spectrum of a Cellular Channel.

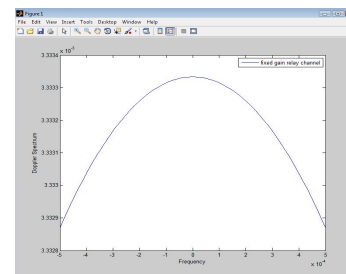


Fig.4: Doppler Spectrum of a Fixed Gain Relay Channel.

The Doppler spectra are more concentrated near zero frequencies for a fixed gain relay channel. This is due to the mobile to-mobile link, where more frequencies will be at or near zero values when the two MS move with identical velocity vectors.

Level Crossing Rate (LCR)

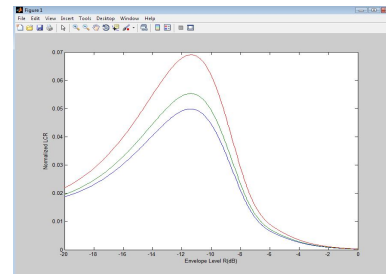


Fig.5: Level Crossing Rate of a Mobile-to-Mobile Channel

LCR is found to increase as the parameter α increases due to increased mobility in the channel.

7. Conclusion

This paper has presented a comprehensive analysis of the statistical properties of A & F relay fading channels. Several properties such as the auto-correlation, LCRs, were studied and verified by means of simulation models. This analysis should prove useful for the design and simulation of several future communication systems that employ A & F relay capability to enhance system performance and capacity.

8. References

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