Synthesis of unequally spaced linear antenna array using Modified Cat Swarm Optimization to suppress Sidelobe levels

Prasanna Kumar K¹, Lakshman Pappula², V S V Prabhakar³
¹Department of Electronics and Communication Engineering, KoneruLakshmaiah Education Foundation, Guntur (Andhra Pradesh), INDIA, prasannakumar@kluniversity.in
²Department of Electronics and Communication Engineering, KoneruLakshmaiah Education Foundation, Guntur (Andhra Pradesh), INDIA, lakshman.pappula@kluniversity.in
³Department of Electronics and Communication Engineering, KoneruLakshmaiah Education Foundation, Guntur (Andhra Pradesh), INDIA, prabhakarvsv@kluniversity.in

ABSTRACT

In this paper a modified cat swarm optimization (MCSO) is introduced that attributes effective global search capabilities with fast convergence. Gaussian mutation is introduced in position updated equation of cat swarm optimization (CSO). The Gaussian mutation allows the CSO algorithm to search their positions in directions to prevent premature convergence and local optima issues. To demonstrate the effectiveness of proposed method, we have applied MCSO to standard complex benchmark mathematical problems. To minimize peak sidelobe level and to control null positions, MCSO is applied to the synthesis of linear aperiodic arrays to optimize positions of antenna elements. Several synthesis examples are considered and the obtained results are compared with linear aperiodic array designs. The obtained numerical results demonstrate that the proposed method is superior to existing methods in terms of accuracy and convergence speed along with minimized side lobe levels.

Key words : Wireless communications, Peak sidelobe level, First Null Beam width, Cat swarm optimization.

1. INTRODUCTION

Most of the antenna arrays[1]-[9] are commonly used for mobile, satellite, radar and wireless communication systems. By creating signal quality, enhancing directivity, extending spectrum efficiency and spreading system coverage, system achievement can be significantly enhanced with the assistance of these antenna arrays. To prevent intervention with other devices which are operating in the same frequency band, these systems need to maintain a minimum peak side lobe level (PSLL). In linear array geometry, there are two methods to obtain low PSLL, one by implementing non-periodic position of the antenna elements and other by thinning of antenna elements. By moving the antenna elements geometric positions from a periodic array, unequally spaced arrays can be generated as shown in Figure 1.

But, synthesis of unequally spaced arrays offers difficult challenges for antenna engineers. The challenges mainly come from the non-linear and non-convex dependence of the array factor on element positions and phases of excitation. The conditions imposed on element positions also increase the difficulty of synthesis.

Figure 1: Illustrating the geometry of unequally spaced arrays

To synthesize unequally spaced array various analytical and nature inspired techniques have been proposed. Most of these methods are based on certain characteristics and behavior of biological, molecular, swarm of insects, and neurobiological systems which require only the function values (and not the derivatives). In recent years, electromagnetic design problems can be successfully optimized by evolutionary algorithms[10] such as genetic algorithm (GA)[11]-[16], differential evolution(DE)[17][18], cat swarm optimization [19]-[23], particle swarm optimization (PSO)[24]-[30] and ant colony optimization(ACO)[31]-[34], Grey Wolf optimization(GWO)[35]. The competency of looking for a global solution to the issues of electromagnetic optimization have been shown by the above mentioned evolutionary algorithms. But most of them lacked in producing low PSLL in array synthesis.

In general, unequally spaced arrays are broadly divided into thinned arrays and non-uniform arrays. In a thinned array, a binary string representing the on/off status of all elements in an equally spaced array has to be determined to obtain lower SLL, hence the actual
number of elements is not fixed. On the other hand, in a non-uniform array, the number of elements is fixed and the element positions are optimized in terms of real vector.

In 2007, a high-performance computing method inspired by cat’s natural behaviour called CSO was introduced by Chu and Tsai [22]. The classical CSO suffers from local optima due to the arbitrary mutation method and does suffer premature convergence while updating the position of the cat. Hence Gaussian mutation based MCSO has been proposed.

2.A NOVEL MODIFIED CSO WITH GAUSSIAN MUTATION STRATEGY

2.1. Traditional CSO

This CSO is designed by defining particular features of the nature of a cat. Depending on the mixture ratio (MR) method, cats are allocated in these two modes.

2.1.1. Seeking mode (SM)

The cat is prepared to move to the next place when in seeking mode (SM), while being alert in the resting place. From the rest place, the motion is so slow that can be calculated by observing the neighboring region. A few important aspects of this mode are:

- Seeking range of the selected dimension (SRD): The SRD specifies the amount of range available for a selected dimension.
- Counts of dimensions to change (CDC): The CDC specifies the number of dimensions to be mutated.
- Seeking memory pool (SMP): The SMP specifies the number of copies of cats to be produced for mutation.

2.1.2. Tracing mode (TM)

In the Tracing mode, cats change their positions very quickly by tracing the targets. Change in the cat’s situation is represented mathematically by the busy hunt. The steps in this mode are as follows:

1. The position and velocity of the \(i^{th}\) cat is defined in the D-dimensional real valued solution space as

\[
X_i^g = [X_{ij}] \text{ where } j = 1 \ldots D \tag{1}
\]

\[
V_i^g = [V_{ij}] \text{ where } j = 1 \ldots D \tag{2}
\]

2. Update the position and velocity of ith cat for every dimension as below

\[
V_{ij}^{g+1} = \omega \cdot V_{ij}^g + C.r. (X_{gbest} - X_{ij}^g) \tag{3}
\]

\[
X_{ij}^{g+1} = X_{ij}^g + V_{ij}^{g+1} \tag{4}
\]

Where \(g\) is the number of the generation, \(i\) is the index of a cat in a swarm, \(j\) is the index of the cat’s position, \(V_{ij}^g\) is the velocity of the ith particle, \(C\) is the acceleration coefficient, \(r \in [0,1]\) is the random number, \(x\) is the inertia weight and the best location of the cat is given by \(X_{gbest}\).

(3) After that the tracing mode cats’ fitness is assessed. If the required solution is not obtained on the basis of the flags, the updated cats will be moved to their modes and this process is repeated until the required solution is achieved. But in Seeking Mode process, arbitrary seeking around the parent cat is being led by the random mutation process. Due to inefficient pursuit of the cat’s place in the neighborhood, this mutation strategy leads to premature convergence.

**Figure 2:** steps involved in the modified CSO algorithm
2.1.3. Algorithm Description of MCSO

1) In D-dimensional solution space, a finite amount of cats are to be initialized randomly.
2) The velocity of the cats is also to be initialized.
3) Then the fitness value of each is to be calculated and the best fitness cat is to be picked and the appropriate position of the cat is stored in the memory as $X_{gbest}$.
4) The cats are shifted to the altered seeking mode and tracing mode depending on their flags, according to MR. In turn, if the cat's flag is set to SM, the cat will be transmitted to the altered SM, otherwise the TM will be implemented.
5) Assess each altered cat's fitness after two modes have been completed and store the best position of the cat as $X_{i,j}$.
6) $X_{gbest} \& X_{i,j}$ fitness values should be correlated and the best position is updated as $X_{gbest}$.
7) Terminate the program, if the required solution is obtained or else repeat the steps from 4 to 7, by continuing the updated cats in the appropriate modes.

The steps involved in the modified CSO algorithm are shown in Figure 2.

The complete details of traditional CSO is discussed in the research article[22]. In seeking mode the position updation is formulated with random mutation strategy. This strategy leads to premature convergence and low solution accuracy.

2.2. Modified CSO

![Gaussian density function with various standard deviations](image)

For various standard deviations the Gaussian distribution curves are shown in Figure 3. From Figure 3, The standard deviation ($\sigma$) and mean ($\mu$) of Gaussian distribution density function is mentioned as

$$f_{normal}(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (5)$$

According to Gaussian law (Eq. (5), the Gaussian random number ($G$) is provided as

$$G(\mu, \sigma^2) = \mu + \sigma G(0,1) \quad (6)$$

Where $G(0, 1)$ is the Gaussian random number usually distributed with standard deviation of 1 and zero mean. From Figure 3 it is evident that, both larger and smaller mutation values can be produced from the Standard deviation value of 1 from other standard deviation values.

A mutant individual ($x^m_i$) is generated by Gaussian mutation following the formula below

$$x^m_i = x_i + N(0, \sigma^2) \quad \text{for} \quad \sigma > 0 \quad (7)$$

$$x^m_i = x_i + \sigma \times N(0,1) \quad (8)$$

where unmutated individual is given by $x_i$. Here, $\sigma$ is conveyed as chosen dimension's mutated value. The position of each dimension of $i^{th}$ cat is therefore amended as

$$x^m_i = x_i + (\text{SRD} \times x_i \times N(0,1)) \quad (9)$$

3. RESULTS AND DISCUSSION

In order to demonstrate the proposed MCSO, we have applied to standard mathematical benchmark multimodal problems. The two functions which we have considered includes Rastrigin($f_1$) and Griewank ($f_2$) as mentioned in Table 1. These are widely adopted to test the performance of new algorithms. It has been observed from Table 2 that, MCSO is achieving acceptable solution faster than CSO and PSO. For example, if we consider 100-D Rastrigin function ($f_1$) MCSO requires 3520 average number of FEAs whereas CSO requires 120900 average number of FEAs to reach the acceptable solution accuracy of $1 \times 10^{-6}$. For the case of 30-D Griewank function($f_2$), MCSO requires 2715 average number of FEAs whereas CSO requires 194740 average number of FEAs.

<table>
<thead>
<tr>
<th>Function</th>
<th>$f$</th>
<th>Benchmark Test Function</th>
<th>$x^*$</th>
<th>$f(x^*)$</th>
<th>Search Range</th>
<th>Acceptable Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rastrigin</td>
<td>$f_1(x)$</td>
<td>$10d + \sum_{i=1}^{D} [x_i^2 - 10\cos(2\pi x_i)]$</td>
<td>(0,0,...,0)</td>
<td>0</td>
<td>[-5.12,5.12]$^{D}$</td>
<td>$1e - 06$</td>
</tr>
<tr>
<td>Griewank</td>
<td>$f_2(x)$</td>
<td>$\sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$</td>
<td>(0,0,...,0)</td>
<td>0</td>
<td>[-600,600]$^{D}$</td>
<td>$1e - 06$</td>
</tr>
</tbody>
</table>
Table 2: Comparison of Solution Accuracy, FEA and Average CPU time for MCSO, CSO and PSO algorithms.

<table>
<thead>
<tr>
<th>Function</th>
<th>30-D</th>
<th>100-D</th>
<th>1000-D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MCS O</td>
<td>PSO</td>
<td>CSO</td>
</tr>
<tr>
<td>Solution Accuracy</td>
<td>0±0</td>
<td>0.35e+00</td>
<td>±0.89e+00</td>
</tr>
<tr>
<td>FEA</td>
<td>3572</td>
<td>X</td>
<td>69550</td>
</tr>
<tr>
<td>Time(sec)</td>
<td>0.1267</td>
<td>X</td>
<td>4.64</td>
</tr>
<tr>
<td>Solution Accuracy</td>
<td>0±0</td>
<td>0.51e+00</td>
<td>±0.18e+00</td>
</tr>
<tr>
<td>FEA</td>
<td>2715</td>
<td>X</td>
<td>71760</td>
</tr>
<tr>
<td>Time(sec)</td>
<td>0.1783</td>
<td>X</td>
<td>8.24</td>
</tr>
</tbody>
</table>

The best results among all the algorithms are marked in bold.
X indicates no trials reached the acceptable solution by the algorithm.

FEA: It is being calculated as the average number of function evaluations is required to reach an acceptable solution over successful runs.

The experimental results for the Rastrigin function($f_1$) and Griewank function($f_2$) shows the robustness of the proposed algorithm by achieving the global optimum zero with accelerated convergence. The evolutionary progress of Rastrigin and Griewank functions for 30-dimensions using MCSO is plotted in Figure 4 and Figure 5 respectively. The algorithm is executed 10 times and the best solution is chosen for all the examples to demonstrate the effectiveness of the proposed approach. All the experiments are conducted using MATLAB with an Intel(R) core(TM) i5-7200U processor operating at 2.71GHz, 8 GB RAM and Windows 10 operating system.

Figure 4: Evolutionary performance of Rastrigin function ($f_1$) for 30-dimensions using MCSO

Figure 5: Evolutionary performance of Griewank function ($f_2$) for 30-dimensions using MCSO

4. PROBLEM FORMULATION

The array factor for even and odd element linear antenna array with N elements is given by

$$ AF (X, \theta) = \sum_{n=1}^{N} \cos[kX_n \cos(\theta)] ; M = 2N \text{ (even)} $$

(10)

$$ AF (X, \theta) = \sum_{n=1}^{N+1} \cos[kX_n \cos(\theta)] ; M = 2N + 1 \text{ (odd)} $$

(11)

where the azimuthal angle is given as $\theta$, the nth element position is given as $X_n$, the wave number is given by $k = 2\pi/\lambda$ and the wavelength is $\lambda$. 

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To suppress the PSLL, the fitness function is formulated as

\[ F(X) = \max\left(\frac{|AF(X,\theta_o)|}{|AF_{\text{max}}|}\right) \quad (12) \]

where \( X = (X_1, X_2, \ldots, X_N) \) is the element position vector, \( \theta_o \) is defined as the angular region excluding the main lobe. The main peak of the pattern is \( AF_{\text{max}} \).

### 4.1. 32 Element Linear Array

In the first example, a 32 element array is synthesized to achieve minimum PSLL. Convergence characteristics using MCSO algorithm for 10 independent runs is shown in Figure 6. The normalized array pattern obtained using the MCSO algorithm along with uniformly illuminated periodic array is shown in Figure 7.

![Figure 6: Convergence plot of the fitness value of the 32 element array using MCSO](image)

![Figure 7: The normalized array pattern of 32-element linear array optimized using MCSO](image)

### 4.2. 37 Element Linear Array

In this second example, a 37 element array is synthesized to achieve minimum PSLL. Convergence characteristics using MCSO algorithm for 10 independent runs is shown in Figure 8. The normalized array pattern obtained using the MCSO algorithm along with uniformly illuminated periodic array is shown in Figure 9.

![Figure 8: Convergence plot of the fitness value of the 37 element array using MCSO](image)

![Figure 9: The normalized array pattern of 37-element linear array optimized using MCSO](image)

Table 3 shows the MCSO optimized element positions, obtained PSLL and FNBW for 32 and 37 elements. It is also seen from Table 3 that the solution for 32 and 37 elements achieved a PSLL of -24.3606dB and -24.7688dB respectively and FNBW of 94.5° and 94.25° respectively. A comparison of the PSLL obtained using DE, simple inversion algorithm(SIA), modified genetic algorithm(MGA) and MCSO algorithms is shown in Table 4. The best PSLL for 32 element linear array in 10 runs was found to be -24.3606 and for 37 element linear array it is found to be -24.7688. From the obtained results it is proved that MCSO outperforms in achieving low PSLL for 32 and 37 elements.
Table 3: Indicating positions, PSLL and FNBW for 32 and 37 Elements.

<table>
<thead>
<tr>
<th>S No</th>
<th>No. of elements in an array</th>
<th>Positions</th>
<th>PSLL</th>
<th>FNBW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32</td>
<td>[1.358511702,0.648347207,3.852316185,1.925747759,0.121350028,1.550879276,1.072001552,2.14896701,0.295583437,4.44354628]</td>
<td>-24.3606</td>
<td>94.5</td>
</tr>
<tr>
<td>2</td>
<td>37</td>
<td>[0.305046105,2.183168386,1.952979103,0.783703475,0.586544818,4.373873931,0.110998322,0.088030817,0.813626389,3.789800395,5.150637651,2.116827009,1.038288527,0.296634944,2.646142724,1.632130183,0.53263506,1.414451487]</td>
<td>-24.7688</td>
<td>94.25</td>
</tr>
</tbody>
</table>

Table 4: Comparison of PSLL for 32 and 37 elements with literature.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>32 Elements</td>
<td>-24.3606</td>
<td>-22.65</td>
<td>-22.29</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The best results among all the algorithms are marked in bold.

4.3. Observation

It can be seen from Table 3 that the proposed MCSO method outperforms the existing methods in terms of PSLL. For 32 element linear array, MCSO produces PSLL of -24.3606dB whereas DE produces -22.65dB and -22.29dB respectively. For 37 element linear array, MCSO produces PSLL of -24.7688dB whereas DE, SIA, MGA produces -22.62dB, -19.37dB and -20.49dB respectively.

It can be seen from Figures 6 and 8 that the standard deviation of the final solution for all the independent runs is minimal. It shows the reliability of the proposed MCSO algorithm to apply to various engineering problems.

5. CONCLUSION

A modified CSO that features global search capability and fast convergence rate has been developed by adopting Gaussian mutation. The better balance of exploration and exploitation has been achieved by using Gaussian mutation in position updated equation. The performance of MCSO has been evaluated with numerical experiments on two complex multimodal functions. The MCSO outperforms traditional CSO,PSO and GA in terms of solution accuracy and convergence rate. The MCSO is applied to unequally spaced antenna array synthesis of 32 and 37 elements to suppress PSLL by optimizing positions between antenna elements.

Numerical results illustrated that MCSO outperforms traditional as well as the modified algorithms in terms of low PSLL. The proposed method may apply to other engineering optimization problems because of its search capabilities.

REFERENCES

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