On The Simulation of Beck Column through a Simple Xiong-Wang-Tabarrok Experimental Model of Centripetally Loaded Column

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ABSTRACT

The controversial articles authored by Koiter and by Sugiyama, Langthjem and Ryu on unrealistic and realistic follower forces indicate the complexity in experimental verification. Elishakoff has made a review on various testing procedures creating the follower force. Xiong, Wang and Tabarrok have proposed a simple experimental model of centripetally loaded column simulating the Beck column. They have not conducted experiments up to the initiation of instability. They have employed a curve-fitting procedure for the measured data relevant to the load parameter and the frequency parameter. The estimated stability load of Beck’s column from the fitted curve is below 10% of the theoretical value. They have analyzed the dynamic characteristics of Beck column and a centripetally loaded column for showing the equivalence of a frequency and the corresponding mode. This article demonstrates their equivalence directly from the dynamic characteristics of Beck column. The discrepancy between dynamic analysis results and test data (if any) can be due to the assigned values of the Young’s modulus and mass density of the material. The characteristic equation of Beck’s column should not be modified to account the discrepancies as being done by Xiong, Wang and Tabarrok, whose experiments are partially successful.

Key words: Beck column, tip-concentrated tangential load, tip-angle, critical load parameter, frequency parameter, coalescence frequency parameter.

1. INTRODUCTION

Dynamic stability of elastic structures is a fascinating topic. After Beck in 1952, many researchers have examined theoretically considering a cantilever column under a tip-concentrated tangential load (the so-called Beck column) [1-13]. The column stability is assessed by generating the load versus frequency curve, namely the eigencurve. Timoshenko and Gere [1] have emphasized experimental verification. Willems [14] has adopted a simple procedure to perform experiments, whose validity is discussed by Huang et al. [15] and other Professors (Augusti, Levinson, Roorda and Herrmann) [16]. The controversial articles of Koiter [17] and Sugiyama et al. [18] on unrealistic and realistic follower forces remains a matter of debate [19]. The basic problem is in the practical realization of follower forces [20, 21]. Mullagulov [22] has created follower forces and tested several hardened steel cantilever rods. Elishakoff [6] has made a review on various testing procedures creating the follower force. Sugiyama et al. [23-25] have conducted experiments by mounting a solid rocket motor at a free-end of the cantilever column to generate a tip-concentrated sub-tangential follower force. Their test results cannot be utilized directly for comparison of critical load estimates [26]. Xiong- Wang-Tabarrok experimental model of centripetally loaded column simulates the Beck column [27]. In order to demonstrate the equivalence of a frequency and the corresponding mode, they have analyzed the dynamic characteristics of Beck column and a centripetally loaded column. This article presents a simple mathematical formulation to show their equivalence directly from the dynamic characteristics of Beck column.

2. MATHEMATICAL FORMULATION

Figure 1 shows a cantilever column under a tip-concentrated follower load \( P \) having tip-angle \( \phi(0) \), the deformed coordinates \( (X, Y) = \int_0^L (\cos \phi, \sin \phi) ds \), and the tip-coordinates \( (X_a, Y_a) \) at \( s = \delta \). \( \delta \) is the length of the deflection curve measured from the tip. \( OB=\delta \), is the column length. \( m \) is the mass per unit length of the column. \( \phi(s) \) is the angle between the tangent to the deformed column and its vertical axis. \( BQ=\delta \), is the distance from the tip (B) of the undeformed column to the point (Q) where the tangent line AQ at the free end of the deformed column intersects the column.
axis OB at Q. Denoting $E$ and $I$ as the Young’s modulus and the moment of inertia respectively, Mutyalarao et al. [13, 26] have presented a system of nonlinear differential equations assuming harmonic motion for large deflections of a cantilever column based on the moment ($M$)-curvature ($\rho^{-1}$) relationship. They have defined $x = \frac{X}{L}$;

\[ y = \frac{Y}{L}; \quad \eta = \frac{s}{L}; \quad \lambda = \frac{PL^2}{EI}, \] is the load parameter;

\[ \omega = \Omega L^2 \left(\frac{m}{EI}\right), \] is the frequency parameter;

\[ \delta = \frac{Y}{L} \quad \text{at} \quad \phi = \phi(0). \]

\[ \frac{\partial^2 \tilde{y}}{\partial \eta^2} + \lambda \tilde{y} - \omega^2 \tilde{y} = 0 \quad \text{(1)} \]

The boundary conditions are

\[ \tilde{y}' = -1, \quad \tilde{y}'' = \tilde{y}''' = 0 \quad \text{at} \quad \eta = 0 \]

\[ \tilde{y} = \tilde{y}' = 0 \quad \text{at} \quad \eta = 1 \]

\[ \text{The solution of the equation (1) is} \]

\[ \tilde{y} = \frac{\psi_2(\eta)}{\psi_1(\eta)} \quad \text{at} \quad \tilde{y} = \tilde{y}' = 0 \]

\[ \text{Equation (4) is solved for the frequency parameter} (\omega) \quad \text{by specifying the load parameter} (\lambda) \quad \text{using the Mathematica}^\text{®}. \]

\[ \frac{\delta}{L} = \frac{\tilde{y}(0)}{\tilde{y}'(0)} = \frac{\psi_2(1)}{\lambda \psi_1(1)} \quad \text{(10)} \]

Stability of the column is assessed from the load versus frequency curve (which is nothing but the eigencurve). Critical load is the minimum load at which the eigencurve cuts the load axis. The dynamic stability load is the minimum load where two branches of eigencurve coalesce. To generate the eigencurves from the first two frequency parameters ($\omega_1$ and $\omega_2$) specifying the load parameter ($\lambda$), the procedure is as follows. By setting $\lambda = 0$, $\omega_1$ and $\omega_2$ are found for the unloaded column from equation (9). The eigencurves are generated considering the first two frequencies by specifying the values of $\lambda$ varying from 0 in steps of 1. It is observed that when $\lambda$ value is reached to 21, Mathematica® provides bifurcated frequency values. Each time, the step size is reduced to half for obtaining the frequency values prior to the bifurcation load parameter. At $\lambda_\text{c} = 20.0509$, the two positive frequency values are tending to the coalescing frequency parameter ($\omega_\text{c}$) value of 11.011.

Xiong et al. [27] have considered a centripetally loaded model and simulated Beck’s column showing equivalence of the first and second modes individually. The centripetal load is applied to three aluminum specimens (having Young’s modulus=69.8 GPa and mass density, $\rho = 2800 \text{ kg/m}^3$) by means of thin steel wires passing through a fixed point at a distance from free end. The present analysis results in Figures 2 to 4 are reasonably in good agreement with measured vibration frequencies [27]. Xiong et al. [27] have not conducted experiments up to stability load. They have employed a curve-fitting procedure for the measured data relevant to the load parameter $\lambda$ and the frequency parameter $\omega$. The stability load of Beck’s column estimated from the fitted curve is found to be below 10% of the theoretical values. It should be noted that the discrepancy between dynamic analysis results and test data can be expected mainly due to the assigned values of the Young’s modulus and mass density of the material. The characteristic equation of Beck’s column should not be adjusted to account the discrepancies as being done in [27].

**Figure 1**: Deformation of a cantilever column under a tip-concentrated follower load ($P$) having tip-angle ($\phi(0)$).

**Figure 2**: Comparison of analysis results and experimental results of Xiong et al. [27] for the aluminum column of size $299.94 \times 10.21 \times 2.85$mm
EXPERIMENTS up to 4.

REFERENCES

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3. CONCLUSION

A centripetally loaded model of Xiong et al. [27] and their simulations on Beck’s column show equivalence of the first and second modes individually. They have analyzed the dynamic characteristics of Beck column and a centripetally loaded column. This article demonstrates their equivalence directly from the dynamic characteristics of Beck column. Test results of Xiong et al. [27] match well with the present analysis results. Xiong et al. [27] have not conducted experiments up to stability load. Instead of correcting the Young’s modulus and the density of the column material, they have wrongly updated the characteristic equation of the Beck column employing a curve-fitting procedure for the measured load parameter ( ) and the frequency parameter ( ).

4. DISCUSSION

A discussion is made on the numerical and experimental results presented in this article. The comparison is made with other existing studies. The results show good agreement with the present study. The dynamic characteristics of Beck column are investigated using a centripetally loaded model. The results show that the dynamic characteristics of Beck column are sensitive to the applied load and the column material. The present study can be extended to investigate the dynamic characteristics of Beck column under other loading conditions.