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The Complexity of Corona Product of Chained Graph and an Outer planar Graph

Fouad Yakoubi[1], Noussaima EL Khattabi[2], Mohamed El Marraki[3]
[1&3]Lrit associated unit the CNRST(URAC29), Faculty of Sciences, University of Mohammed V, Morocco,
fouad.yakoubii@gmail.com & marraki@fsr.ac.ma
[2]Laboratory of Conception and Systems, Faculty of Sciences, University of Mohammed V, Morocco,
Khattabi@fsr.ac.ma

ABSTRACT

Computing the spanning trees number in a graph is the most important studied problems in graph theory. Matrix tree theorem was the first method computing this graph parameter, which has been inefficient. That why computing this number remains a challenge, especially for large graph such as complex networks. For this reason we developed a combinatorial method computing the number of spanning trees in a families graph. In this work, we aimed to compute the number of spanning trees in the corona product graph of a chain planar graphs and an Outer planar graph, using our recursive method to investigate an explicit computing this number.

Key words: Corona product, Complex system, Graph Complexity, Liner chain graphs, Spanning tree.

1.INTRODUCTION

Computing the number of spanning trees of a graph G is the most studied problems in graph theory. So it has been an important invariant since its appearance. This number is very useful in practical areas, such as designing electrical circuits, estimating the network reliability [7-9],[11],[15]. The number of spanning trees in a graph G denoted by $\tau(G)$, also called the complexity of G. The matrix tree theorem [9], is the first method computing the number of spanning trees of G, which express this number as the co-factor matrix determinant of the Laplacian matrix of G. However, this calculation is very tedious for large graphs. For this reason many works have investigated several approaches to derive recursive methods computing the complexity of some graphs families [3-5],[12-14]. In this work we study the corona product of a planar graph with one articulation vertex G_n and an outer planar graph G_m. A planar graph G is a graph with one articulation vertex, when it is formed of two sub graphs G_1 and G_2 that have only one common vertex $G = G_1 \bullet G_2$, see figure 1(a). The corona product graph of a planar graph G_n and an outer planar graph G_m. denoted by $G_n \lozenge G_m$, is the graph obtained by taking n copies of G_m, and joining the i^{th} vertex of G_n to each vertex in the j^{th} copy of G_m (see figure 3). In order to compute the complexity of corona product of some special graphs, an algebraic method -chebyshev polynomials method- was used in [6]. However, this method require tedious algebraic calculation. In this paper, we suggest a combinatorial method based on a recursive method calculating the number of spanning trees in corona product graph of a graph with one articulation vertex $G_{n_1} \bullet G_{n_2}$ and an outer planar graph G_{n_3}. Then, we treat the case of linear chain of k planar graphs $G = G_{n_1} \bullet G_{n_2} \bullet G_{n_3} \bullet\bullet\bullet G_{n_k}$ giving an explicit formula computes the number of spanning trees in this corona product graph of a closed or linear chain (see figures 1(b) and (c), and an outer planar graph G_q.

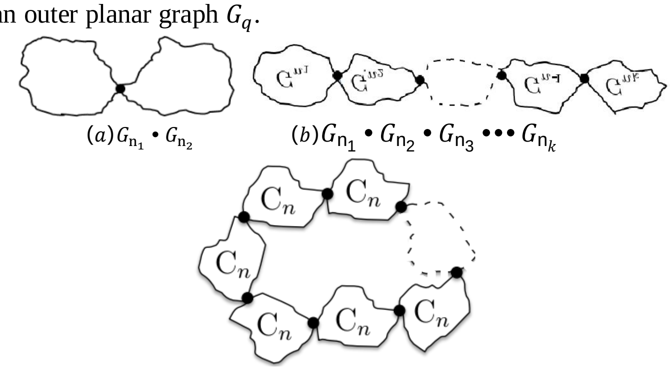


Figure 1: Linear and closed chain graphs

2.PRELIMINARY

In this section we introduce the theorems that will be useful in our work:

Let G_n be a linear chain formed of k graphs as is shown in figure 1(b) .The number of spanning trees in G_n is given by[4]:

$$\tau(G_{n1} \bullet G_{n2} \bullet G_{n3} \bullet\bullet\bullet G_{nk}) = \prod_{i=1}^{k} \tau(G_{n_i}) \qquad (1)$$

Let G_n be a planar graph and G_m an outer planar graph. The complexity of the corona product $G_n \lozenge G_m$ is given by [2]:

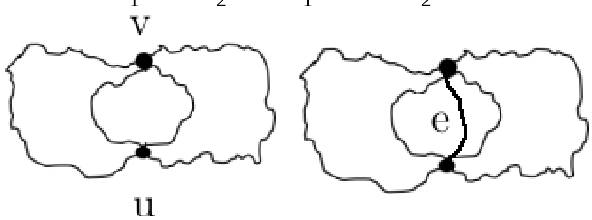
$$\tau(G_n \lozenge G_m) = \tau(G_n) \times (\tau(G_m \lozenge P_1))^n$$

If G is a graph formed of two sub graphs G_1 and G_2 ($G = G_1 : G_2$) that are crossed only in two vertex u and v as illustrated in Figure 2(a). Then the complexity of G is given as follows [8]:

$$\tau(G)=\tau(G_1)\times\tau(G_2.uv)+\tau(G_1.uv)\times\tau(G_2) \qquad (3)$$

If G is a graph formed of two adjacent sub graphs G_1 and G_2 $G = G_1 | G_2$, and they have an edge in common e as illustrated in figure 2(b). Then the complexity of G is given by [8]:

$$\tau(G)=\tau(G_1)\times\tau(G_2)-\tau(G_1-e)\times\tau(G_2-e) \qquad (4)$$



(a) G_1:G_2 (b) $G_1|G_2$

Figure 2: planar graphs G_1:G_2 and $G_1|G_2$

3.MAIN RESULTS

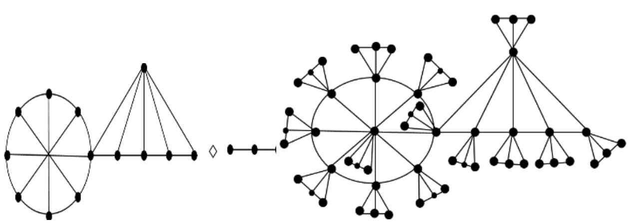
In this paper, we propose a new method for computing the number of spanning trees in corona product graph of a planar graph with one articulation vertex and chain graphs with an outer planar graph.

3.1 Case of planar graph with one articulation *vertex.*

Lemma 3.1:

Let $G_n \bullet G_m$ be a graph with one articulation vertex and G_q an outer planar graph, the number of spanning trees in corona product of $G_n \bullet G_m$ and G_q is given by:

$$\tau((G_n \bullet G_m)\diamond G_q)=\tau(G_n)\times\tau(G_m)\times\tau(G_q\diamond P_1)^{n+m-1}$$



(a) $W_m \bullet F_n$ and P_q (b) $(W_n \bullet F_m)\diamond P_q$

Figure 3: The Corona product graph $(W_n \bullet F_m)\diamond P_q$

Proof: To calculate the complexity of corona product graph of $G_n \bullet G_m$ and G_q, we use equations (1) and (2):

$\tau((G_n \bullet G_m)\diamond G_q)=\tau(G_n \bullet G_m)\times\tau(G_q\diamond P_1)^{n+m-1}$,
and $\tau(G_n \bullet G_m)=\tau(G_n)\times\tau(G_m)$. Hence the result.

Now, we derive an explicit formula counting the number of spanning trees in corona product graph of some families of graphs with one articulation vertex and an outer planar graph.

Let $W_n \bullet F_m$ be a graph with one articulation vertex formed of Wheel graph and Fan graph, and P_q is a simple path and S_q a star tree (see figure 3(a)).

Theorem 3.2:

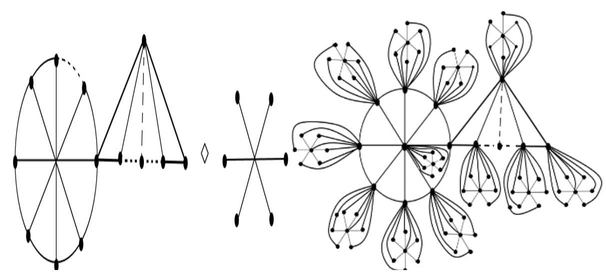
The complexity of corona product graphs $(W_n \bullet F_m) \diamond P_q$ and $(W_n \bullet F_m) \diamond S_q$ are given by:

$$\tau((W_n \bullet F_m) \diamond P_q) = \left(\left(\frac{3+\sqrt{5}}{2}\right)^n - \left(\frac{3-\sqrt{5}}{2}\right)^n - 2\right) \times \left(\frac{1}{\sqrt{5}}\left(\left(\frac{3+\sqrt{5}}{2}\right)^{m-1} - \left(\frac{3-\sqrt{5}}{2}\right)^{m-1}\right)\right) \times \left(\frac{1}{\sqrt{5}}\left(\left(\frac{3+\sqrt{5}}{2}\right)^{n} - \left(\frac{3-\sqrt{5}}{2}\right)^{n}\right)\right)^{n+m-1}$$

$$\tau((W_n \bullet F_m) \diamond S_q) = \left(\left(\frac{3+\sqrt{5}}{2}\right)^n - \left(\frac{3-\sqrt{5}}{2}\right)^n - 2\right) \times \left(\frac{1}{\sqrt{5}}\left(\left(\frac{3+\sqrt{5}}{2}\right)^{m-1} - \left(\frac{3-\sqrt{5}}{2}\right)^{m-1}\right)\right) \times \left((q+1)2^{q-2}\right)^{n+m-1}$$

Proof:

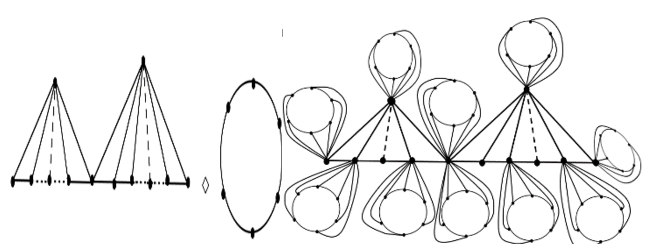
In order to calculate $\tau((W_n \bullet F_m) \diamond P_q)$ and $\tau((W_n \bullet F_m) \diamond S_q)$, we use Lemma 3.1, then we replace $\tau(W_n), \tau(F_n), \tau(W_n)$, $\tau(P_1 \diamond SP_q)$ and $\tau(P_1 \diamond S_q)$ by their values (See [2],[3] ,[5]).



(a) $W_n \bullet F_m$ and S_q (b) $(W_n \bullet F_m) \diamond S_q$

Figure 4: the Corona product graph

Let $F_n \bullet F_m$ be a graph with one articulation vertex formed of two Fan graphs and C_q is a cycle. We compute the number of spanning trees in corona product graph $(F_n \bullet F_m) \diamond C_q$, by using Lemma 3.1 and substituting $\tau(F_n)$and $\tau(P_1 \diamond C_q)$ theirs by values(See [2],[3] ,[5]).



(a) $F_n \bullet F_m$ and C_q $(F_n \bullet F_m) \diamond C_q$

Figure 5: The Corona product graph $(F_n \bullet F_m) \diamond C_q$

Theorem 3.3

The number of spanning trees in $(F_n \bullet F_m) \lozenge C_q$ is given by:

$$\tau((F_n \bullet F_m) \lozenge C_q) = \left(\frac{1}{\sqrt{5}}\left(\left(\frac{3+\sqrt{5}}{2}\right)^{n-1} - \left(\frac{3-\sqrt{5}}{2}\right)^{n-1}\right)\right)$$
$$\times\left(\frac{1}{\sqrt{5}}\left(\left(\frac{3+\sqrt{5}}{2}\right)^{m-1} - \left(\frac{3-\sqrt{5}}{2}\right)^{m-1}\right)\right)$$
$$\times\left(\left(\left(\frac{3+\sqrt{5}}{2}\right)^{q} - \left(\frac{3-\sqrt{5}}{2}\right)^{q} - 2\right)\right)^{n+m-1}$$

In the previous part, we computed the number of spanning trees in corona product graph of a graph with one articulation vertex $G_n \bullet G_m$ and an outer planar graph G_q. To treat the general case. Let $G_{n1} \bullet G_{n2} \bullet G_{n3} \bullet\bullet\bullet G_{nk}$ be a linear chain map, see figure 1(b), and an outer planar graph G_m.

3.2Case of linear chain of planar graphs

Lemma 3.4 :

Let C $(C = G_{n1} \bullet G_{n2} \bullet G_{n3} \bullet\bullet\bullet G_{nk})$ be a linear chain of planar graphs and G_m an outer planar graph, then the complexity of the corona product graph of C and G_m is given by :

$$T((G_{n1} \bullet G_{n2} \bullet G_{n3} \bullet\bullet\bullet G_{nk}) \lozenge G_m) =$$
$$\prod_{i=1}^{k} \tau(G_{n_i}) \times (\tau(P_1 \lozenge G_m))^{(\sum_{i=1}^{k} n_i)-k+1}$$

Proof:

The chain $(G = G_{n1} \bullet G_{n2} \bullet G_{n3} \bullet\bullet\bullet G_{nk})$ has an articulation vertex between each adjacent graphs. Then, there are k-1 shared vertices in total, Let |V(G)| be the total number of vertices in G. So, $|V(G)| = (\sum_{i=1}^{k} n_i) - (k-1)$ Applying Equation (2), we obtain:

$$\tau(G \lozenge G_m) = \tau(G) \times (\tau(P_1 \lozenge G_m))^{(\sum_{i=1}^{k} n_i)-k+1}$$

Using equation (1) $\tau(G) = \prod_{i=1}^{k} \tau(G_{n_i})$, then

$$\tau(G \lozenge G_m) = \prod_{i=1}^{k} \tau(G_{n_i}) \times (\tau(P_1 \lozenge G_m))^{(\sum_{i=1}^{k} n_i)-k+1}$$

Now we compute the Complexity of corona product graph of a linear chain of planar graphs G with a path P_m and a cycle C_m.

Theorem 3.5 :

Let G is a linear chain of k-Fan graphs

$G = F_{n1} \bullet F \bullet F_{n3} \bullet\bullet\bullet F_{nk}$ and P_m is a path, then the number of spanning trees in corona product graph

$(F_{n1} \bullet F \bullet F_{n3} \bullet\bullet\bullet F_{nk}) \lozenge P_m$ is given by :

$$\tau(F_{n1} \bullet F \bullet F_{n3} \bullet\bullet\bullet F_{nk}) \lozenge P_m$$
$$= \prod_{i=1}^{k}\left(\frac{1}{\sqrt{5}}\left(\left(\frac{3+\sqrt{5}}{2}\right)^{n_i-1} - \left(\frac{3-\sqrt{5}}{2}\right)^{n_i-1}\right)\right)$$
$$\times\left(\frac{1}{\sqrt{5}}\left(\left(\frac{3+\sqrt{5}}{2}\right)^{n-1} - \left(\frac{3-\sqrt{5}}{2}\right)^{n-1}\right)\right)^{(\sum_{i=1}^{k} n_i)-k+1}$$

Theorem 3.6 : Let C be a linear chain formed of k cycle graphs $(C = C_n \bullet C_n \bullet\bullet\bullet C_n)$ and G_m. The complexity of the corona product graph $(C_n \bullet C_n \bullet\bullet\bullet C_n) \lozenge G_m$ is given as follows:

$$\tau((C_n \bullet C_n \bullet\bullet\bullet C_n) \lozenge G_m) = n^k \times \tau(P_1 \lozenge G_m)^{nk-k+1}$$

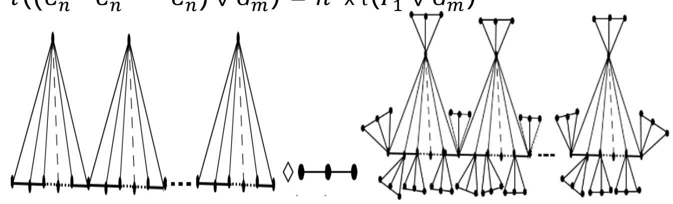


Figure 6 : The Corona product graph $(F_{n1} \bullet F \bullet F_{n3} \bullet\bullet\bullet F_{nk}) \lozenge P_m$

1.1 Case of closed chain of planar graphs

In this part we compute the complexity of corona product graph $\tau(C_{O_k} \lozenge G_m)$ of the closed chain graph C_{O_k} and an outer planar G_m. At First, we need to calculate the complexity of the graph $C_n.uv$, which is the graph obtained from C_n after pasting the vertex u with v (See Figure 7).

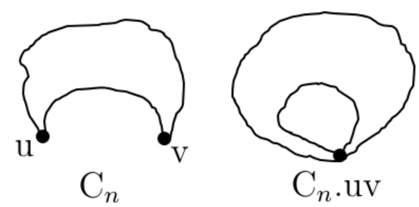


Figure 7: the graphs C_n and $C_n.uv$

The number of spanning trees in the corona product graph $C_n.uv \lozenge G_m$ of the planar graph $C_n.uv$ and the outer planar graph G_m is computed by using equation 2, we get :

$$\tau(C_n.uv \lozenge G_m) = \tau(C_n.uv) \times \tau(G_m \lozenge P_1)^{n-1} \quad (12)$$

Lemma 3.7:

If C_{O_k} is a closed chain formed of k-planar graphs of C_n. Then, the complexity of the corona product graph $C_{O_k} \lozenge G_m$ is given by :

$$\tau(C_{O_k} \lozenge G_m) = k(\tau(C_n) \times \tau(G_m \lozenge P_1)^{n-1})^{k-1}$$
$$\times\, \tau(C_n.uv) \times \tau(G_m \lozenge P_1)^{n-1}$$

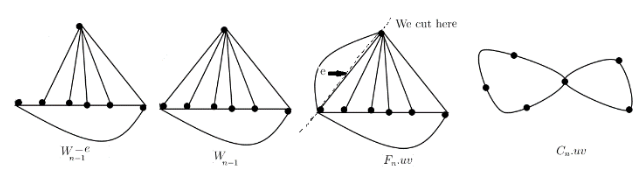


Figure 8: the graphs F_n.uv and C_m.uv

Proof: We denoted by L_k a linear chain formed of k planar graph of C_n. and by H_k the corona product $L_k \lozenge G_m$ of L_k and G_m. To prove the previous lemma, we used the contraction method, choosing two vertex u and v in the map C_{O_k}, and applying equation 3 and lemma 3.4 we obtain :

$$\tau(C_{O_k} \lozenge G_m) = \tau(H_{k-1}) \mathrm{x} \tau(C_n.uv \lozenge G_m) + \tau(C_{O_{k-1}} \lozenge G_m) \mathrm{x} \tau(C_n \lozenge G_m)$$

And $\tau(H_{k-1}) = (\tau(C_n))^{k-1} \mathrm{x}\, \tau(P_1 \lozenge G_m)^{n(k-1)}$,

by using the same formulas again and again, we get:

$$\tau(C_{O_k} \lozenge G_m) = (\tau(C_n))^{k-1} \mathrm{x}\, \tau(P_1 \lozenge G_m)^{n(k-1)} \mathrm{x} \tau(C_n.uv)$$

$$\mathrm{x}\, \tau(G_m \lozenge \mathrm{P}_1)^{n-1} + \tau(C_{O_{k-1}} \lozenge G_m) \mathrm{x} \tau(C_n \lozenge G_m)$$

$$\tau(C_{O_{k-1}} \lozenge G_m) = (\tau(C_n))^{k-2} \mathrm{x}\, \tau(P_1 \lozenge G_m)^{n(k-2)} \mathrm{x} \tau(C_n.uv)$$

$$\mathrm{x}\, \tau(G_m \lozenge \mathrm{P}_1)^{n-1} + \tau(C_{O_{k-1}} \lozenge G_m) \mathrm{x} \tau(C_n \lozenge G_m)$$

$$\tau(C_{O_{k-2}} \lozenge G_m) = (\tau(C_n))^{k-3} \mathrm{x}\, \tau(P_1 \lozenge G_m)^{n(k-3)} \mathrm{x} \tau(C_n.uv)$$

$$\mathrm{x}\, \tau(G_m \lozenge \mathrm{P}_1)^{n-1} + \tau(C_{O_{k-1}} \lozenge G_m) \mathrm{x} \tau(C_n \lozenge G_m)$$

$$\vdots$$

$$\tau(C_{O_3} \lozenge G_m) = (\tau(C_n))^{2} \mathrm{x}\, \tau(P_1 \lozenge G_m)^{2n} \mathrm{x} \tau(C_n.uv)$$

$$\mathrm{x}\, \tau(G_m \lozenge \mathrm{P}_1)^{n-1} + \tau(C_{O_{k-1}} \lozenge G_m) \mathrm{x} \tau(C_n \lozenge G_m)$$

$$\tau(C_{O_2} \lozenge G_m) = \tau(C_n) \mathrm{x}\, \tau(P_1 \lozenge G_m)^{n}\, \mathrm{x}\, \tau(C_n.uv)$$

$$\mathrm{x}\, \tau(G_m \lozenge \mathrm{P}_1)^{n-1} + \tau(C_{O_{k-1}} \lozenge G_m) \mathrm{x} \tau(C_n \lozenge G_m)$$

We multiply the first equation by $\tau(C_n) \mathrm{x}\, \tau(P_1 \lozenge G_m)^n$, the second by $(\tau(C_n))^2 \mathrm{x}\, \tau(P_1 \lozenge G_m)^{2n}$ (and so on, until the last equations is multiplied by $(\tau(C_n))^{k-2} \mathrm{x}\, \tau(P_1 \lozenge G_m)^{n(k-2)}$. When we add up all those equations we get the complexity of $C_{O_k} \lozenge G_m$

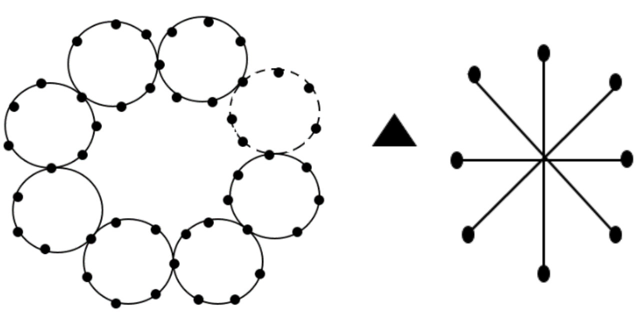


Figure 9:The closed chain cycle graphs C_{O_k} & the Star graph G_m

Corollary 3.8:

If C_{O_k} is a closed chain formed of k-cycles, F_{O_k} is a closed chain formed of k-Fan graphs and S_m a Star graph. Then, the complexity of the corona product graphs $C_{O_k} \lozenge S_m$ and $F_{O_k} \lozenge S_m$ are given as follows:

$$\tau(C_{O_k} \lozenge S_m) = \mathrm{k}\left(n\, \mathrm{x}\, ((m+1)2^{m-2})^{n-1}\right)^{k-1} n_1 n_2\, ((m+1)2^{m-2})^{n-1}$$

$$\tau(F_{O_k} \lozenge S_m) =$$

$$k\left(\frac{1}{\sqrt{5}}\left(\left(\frac{3+\sqrt{5}}{2}\right)^{n-1} - \left(\frac{3-\sqrt{5}}{2}\right)^{n-1}\right) \mathrm{x}\, ((m+1)2^{m-2})^{n-1}\right)^{k-1}$$

$$\mathrm{x}\, \tau(F_n.uv)\, \mathrm{x}\, ((m+1)2^{m-2})^{n-1}$$

Where $\tau(F_n.uv) = 2\left(\left(\frac{3+\sqrt{5}}{2}\right)^{n-1} - \left(\frac{3-\sqrt{5}}{2}\right)^{n-1} - 2\right) -$
$\frac{1}{\sqrt{5}}\left(\left(\frac{3+\sqrt{5}}{2}\right)^{n-2} - \left(\frac{3-\sqrt{5}}{2}\right)^{n-2}\right) - \left(\left(\frac{3+\sqrt{5}}{2}\right)^{n-2} - \left(\frac{3-\sqrt{5}}{2}\right)^{n-2} - 2\right)$

To calculate the complexities of the graphs $\tau(C_{O_k} \lozenge G_m)$ and $\tau(F_{O_k} \lozenge G_m)$, we need to have $C_n.uv$ and $F_n.uv$ (see Figure 8), applying Lemma 3.7. $C_n.uv$ is formed of two sub graphs which have a single articulation vertex, which have n_1, n_2 vertex, respectively. Then $\tau(C_n.uv) = n_1 \mathrm{x} n_2$, with $n = n_1 + n_2 - 1$. To compute $\tau(F_n.uv)$ we choose the edge e of $F_n.uv$ as illustrated in Figure 8, then by using Equation 4 we get:

$\tau(F_n.uv) = 2\tau(W_{n-1}) - \tau(W_{n-1} - e)$ and

$\tau(W_{n-1} - e) = \tau(F_{n-1}) - \tau(W_{n-2})$. []

So, $(F_n.uv) = 2\tau(W_{n-1}) - \tau(F_{n-1}) - \tau(W_{n-2})$.

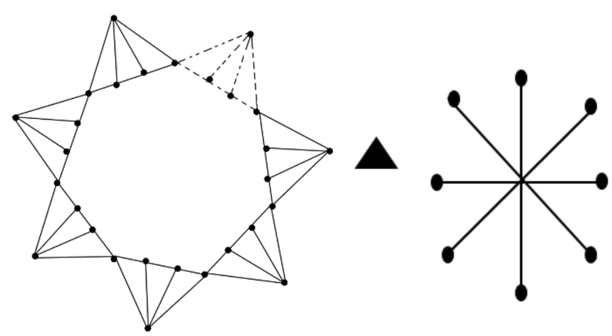


Figure 10: The closed chain of k Fan graphs F_{O_k}, Star graph S_m

4.CONCLUSION

The problem of computing the number of spanning trees is a very important parameter in the analysis and synthesis of reliable networks. Indeed, the number of spanning trees in a graph is a natural measurement of networks reliability. Computing this number using an algebraic method is useless in the practical areas. Investigating an explicit formula to compute this number for various graphs has become a vital role namely in computer science. In this paper, we presented recursive methods for the purpose of providing an explicit formula computing the number of spanning trees in corona product graph of planar graph with one articulation vertex and planar chain graphs with an arbitrary outer planar graph. Then, using our results, we gave the complexity of some corona product graphs, namely, chain of k-Fan graphs with a path and chain of k-cycles with an arbitrary outer planar graph.

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