



## The Method of Hidden Transmitters Detection based on the Differential Transformation Model

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### ABSTRACT

The article investigates the method of calculating the basic parameters of random signals emitted by hidden transmitters. The method is based on a mathematical model of differential signal transformations within the framework of correlation theory. It is proved that under certain initial conditions the expected value of a random signal consists of the sum of all expectations of the differential spectrum. A method for numeric calculating of the variance, which consists of determining the differential spectrum and calculating the variance for each discrete component of the signal, is presented. The approach to determining the correlation function of a random signal is justified. It is shown that on the basis of two differential spectra it consists of the sum of all discrete components of the differential spectrum defined for the correlation function. As a partial case, the conditions for the coincidence of the correlation function with the variance are given. A model of hidden transmitter was built. In order to obtain its characteristics, a simulation of a random signal was made in the MathLab software environment, which modulates digital signal of hidden transmitter. The numerical parameters of the random signal are obtained: expectation and variance. Three-dimensional graphs of the results of random signal modeling have been constructed, which allows to visualize the implementation of the random signal. Graphical results of the signal itself and its expectation and variance were also obtained. Analysis of the statistical characteristics showed that the expectation completely repeats the graphical appearance of the signal. The variance graph repeats the signal at zero but with increasing time. These results confirm the adequacy of the mathematical model of the hidden transmitters based on the theory of differential transformations.

**Key words:** Hidden transmitter, differential transformations, math model, random signal, expectation, variance.

### 1. INTRODUCTION

Differential transformations are a relatively new operating method, initiated in [1]. This math method, unlike the known integral and discrete transformations, is based on the translation of originals into the image area using differentiation operation. In the mathematical modeling of physical objects and processes described by differential and integral equations, differential transformations allow to replace the operations of integration and differentiation by equivalent algebraic operations, both numerically and analytically.

Mathematical modeling of radio monitoring processes based on differential transformations, characterized in that the monitoring process consists in the conditions of random disturbance. The process of detecting a third party radio signal is also random. Modeling random processes in complex nonlinear systems requires considerable computational resources. In real-time systems, the calculation speed required to obtain the required accuracy of random process modeling may exceed the speed limit that modern computer technology can provide. Therefore, nowadays, the scientific task of developing a math method that will allow modeling the process of detecting random signals on the basis of the model of searching for hidden transmitters in real time and to calculate the main statistical parameters of these random signals is relevant.

### 1.1 Literature analysis and problem statement

An attempt to develop a math method of differential transformations and its application to a class of random or stochastic functions and processes was made in [1]. The math algorithm of differential transformations has been applied to a vector random function that can differentiate the required number of times. This requirement significantly limited the possibility of differential transformations within the local region of the random process intersection for each fixed point in time. However in [2] the described application of differential transformations provides an opportunity to develop only an approximate method for modeling random processes.

The article [3] considers a math method of differential transformations applied to a random function that can differentiate over the entire interval of determining the function. Differential transformation of a random function allows to obtain the energy spectrum of signals, which is subsequently used for signal recognition. But only the theoretical part of the question is considered. The questions of reliability of the obtained results are not considered at all, so it is unsearched how far the obtained results correspond to the initial random function.

The approach proposed in [1] does not allow accurate modeling of random processes. However, this possibility exists because differential transformations refer to exact operating methods [2].

The method of differential transformations is widely used in various fields. For example in paper [4], generalized differential transform method is applied to obtain an approximate solution of linear and nonlinear differential equation of fractional order with boundary conditions. Several numerical examples are considered and comparisons with the existing solution techniques are reported. Results show that the method is effective, easier to implement and very accurate when applied for the solution of fractional boundary values problems.

The paper [5] considers the differential transformation method (DTM) employed to find the semi-analytical solutions of artificial epidemic models for constant population. Firstly, the theoretical background of DTM is studied and followed by constructing the solutions of artificial epidemic models. Furthermore, the convergence analysis of DTM is proven by proposing two theorems. Finally, numerical computations are made and compared with the exact solutions. From the numerical results, the solutions produced by DTM approach the exact solutions which agreed with the proposed theorems. It can be seen that the DTM is an alternative technique to be considered in solving many practical problems involving differential equations.

In the [6] show that the DTM technique is accurate and efficient and require less computational effort in comparison to the other methods. Results revealed that DTM save time

and space and it is easy to implement. In [7] analytical and numerical solutions are determined and the results are compared to that of Runge-Kutta method. The results agreement between DTM and Runge-Kutta method.

The paper [8] shows the method of distributed transformation training patterns vocal sounds to a unified amplitude-time window and method of distributed clustering training patterns for vocal sounds have been proposed for distributed forming reference patterns of speech vocal sounds in the paper. These methods allow fast convert quasiperiodic sections of different lengths to a single amplitude-time window for subsequent comparison and accurately and quickly determine the optimal number of clusters, which increases the probability of clusterization. The proposed methods can be used in recognition systems.

In [9] three phase circuits are investigated. Based on the research conducted in MATLAB, an optimization model was developed to measure the power in three phase circuits. In particular, a contour database of power was developed and optimization algorithms were developed. These algorithms can be used in the development of characteristics of miscellaneous information receipts.

The research made in [10] focuses on the implementation of both a facial recognition system and a facial detection system in MATLAB. This research would use for investigates the method of calculating the basic parameters of random signals emitted by hidden transmitters.

In [11] quantitative methods for evaluating the functional stability of a special purpose wireless sensor network by established parameters are proposed. Based on these estimates, recommendations can be made to extend the structure or make sound requirements for the structure of the wireless sensor network. In addition, network data contains price information and is subject to intrusion by the attackers. Therefore, it is necessary to investigate the methods of hidden removal of information in these networks.

In [12] the model Petri nets are considered and basic models are created on the basis of which they will allow to maintain the functional stability of the information system under the influence of external and internal destabilizing factors. Such models allow to take into account both features of construction of information system, and dynamics of processes of change of its states at functioning under conditions of influence of external and internal destabilizing factors. However, such types of destabilizing factors as the hidden information theft are not explored in that article.

So, the problem of random processes modeling, which allows, on the basis of differential transformations, to obtain accurate models of determining random signals in the area of images within the framework of correlation algorithms, is important and relevant. In spite of application this method for hidden transmitters searching it is strictly important to calculate the main statistical parameters of random signals.

## 1.2 Aim of the article

Digital media typically emit a random signal. To detect and recognize this random signal against the background of legal radio signals, it is necessary to present this signal in a convenient form for digital analysis. Therefore, the purpose of the study is to represent the signal in the form of a math model based on differential transformations, which most accurately reflects the signal, and will allow to determine the statistical characteristics of the signal.

## 2. THE MAIN SECTION

The main statistical parameters of random signals will be considered in the framework of correlation theory. Consider a one-dimensional random signal  $x(t, \delta)$  where  $\delta$  – given a random variable. In the image area, the model of this signal is represented by the differential spectrum  $X(K, \delta)$ . After the conversely perform  $X(K, \delta)$  in the time zone according to [1], we get the expression:

$$x(t, \delta) = \sum_{K=0}^{\infty} \left(\frac{t}{H}\right)^K X(K, \delta) \quad (1)$$

Expected value  $m_x(t)$  of a random signal  $X(K, \delta)$ , described by expression (1), is calculated as follows:

$$m_x(t) = m_x(t) = M[x(t, \delta)] = \sum_{K=0}^{\infty} (t/H)^K M[X(K, \delta)] \quad (2)$$

where

$$M[X(K, \delta)] = m_x(K) = \int_{-\infty}^{\infty} X(K, \omega) p(\omega) d\omega, \quad (3)$$

and  $p(\delta)$  – the probability density distribution of the random variable  $\delta$ .

If the start of a random signal  $x(t, \delta)$  is considered from zero time  $t_0=0$ , then  $H = t - t_0 = t$ . After this the expressions (2)-(3) are simplified:

$$m_x(t) = \sum_{K=0}^{\infty} m_x(K) \quad (4)$$

From expression (4) follows that the expected value  $m_x(t)$  of a random signal  $X(K, \delta)$  is determined by the sum of expected values  $m_x(K)$  of all discrete differential spectrum

$$X(K, \delta), \quad K=0, 1, 2, \dots, i, \quad \text{at } H = t.$$

The variance of the random function  $x(t, \delta)$  is calculated as the expected value of a square of centered random variable:

$$D_{(x)}(t) = M[x^2(t, \delta)] - m_x^2(t) \quad (5)$$

Lets introduce an auxiliary random function that equal to the square of the function under study:

$$U(t, \delta) = x^2(t, \delta) \quad (6)$$

Then expression (5) takes the form:

$$D_{(x)}(t) = M[U(t, \delta)] - m_x^2(t) \quad (7)$$

Let us translate expressions (5) and (6) into the region of images by means of differential transformations:

$$D_{(x)}(K) = M[U(K, \delta)] - m_x^2(K) \quad (8)$$

$$U(K, \delta) = X(K, \delta) * X(K, \delta) = \sum_{e=0}^{e=K} X(K-l, \delta) X(l, \delta) \quad (9)$$

$$M[U(K, \delta)] = m_x(K) = \int_{-\infty}^{\infty} U(K, \delta) p(\delta) d\delta \quad (10)$$

$$m_x^2(K) = m_x(K) * m_x(K) = \sum_{l=0}^{l=K} m_x(K-l) m_x(l) \quad (11)$$

where  $*$  is the symbol of the multiplication operation in the image area.

Expressions (8) - (11) allow us to determine the differentials spectrum  $D_x(K)$ , which models the variance  $D_x(t)$  in the image area. In order to restore the dispersion  $D_x(t)$  in the time zone on the differentials spectrum  $D_x(K)$ , reverse differential transformations of the form should be applied:

$$D_{(x)}(t) = \sum_{K=0}^{\infty} (t/H)^K D_x(K) \quad (12)$$

Given that it is a random signal  $x(\delta, t)$  is being considered by  $t_0=0$ , and  $H = t - t_0 = t$  expression (12) takes the following form:

$$D_{(x)}(t) = \sum_{K=0}^{\infty} D_x(K), \quad (13)$$

where  $D_x(t)$  – the differential spectrum calculated by expressions (7) - (9) on the basis of the differential spectrum  $X(K, \delta)$ . Expression (13) gives a simple algorithm for determining the variance  $D_x(t)$  random signal  $x(t, \delta)$ .

It is now necessary for each discrete differential spectrum  $X(K, \delta)$  find the variance  $D_x(K)$ ,  $K=0, 1, 2, \dots, i$ , of all discrete differentials spectrum  $X(K, \delta)$  at  $H = t$ .

Correlation function  $R_x(t_1, t_2)$  random signal  $x(t, \delta)$  build on the basis of two differential spectra  $X(K, \delta)$  on the basis of direct differential transformations of discrete [1]. We choose two time intervals for calculations  $H_1 = t_1 - t_0$  and  $H_2 = t_2 - t_0$ . Using  $t_0 = 0$ , we received  $H_1 = t_1$  and  $H_2 = t_2$ .

Denote the differential spectrum  $X(K, \delta)$  obtained at  $H_1 = t_1$  as  $X(K, t_1, \delta)$ . For time  $H_2 = t_2$  denote as  $X(K, t_2, \delta)$ .

We determine the expectations of the discrete of these two differential spectra

$$m_x(K, t_1) = M[X(K, t_1, \delta)] = \sum_{l=0}^{l=K} X(K, t_1, \delta) p(\delta) d\delta \quad (14)$$

$$m_x(K, t_2) = M[X(K, t_2, \delta)] = \sum_{l=0}^{l=K} X(K, t_2, \delta) p(\delta) d\delta \quad (15)$$

In the image area, multiply the differential spectra (14) and (15):

$$m_x(K, t_1) * m_x(K, t_2) = \sum_{l=0}^{l=K} m_x(K-l, t_1) m_x(l, t_2) \quad (16)$$

At the same time, the product of two differential spectra should be formed  $X(K, t_1, \delta)$  and  $X(K, t_2, \delta)$  in the image area. Then we get the expression:

$$\begin{aligned} Q(K, t_1, t_2, \delta) &= X(K, t_1, \delta) * X(K, t_2, \delta) = \\ &= \sum_{l=0}^{l=K} X(K-l, t_1, \delta) X(l, t_2, \delta) \end{aligned} \quad (17)$$

Find the expectation of (17):

$$\begin{aligned} m_q(K, t_1, t_2, \delta) &= M[Q(K, t_1, t_2, \delta)] = \\ &= \int_{-\infty}^{\infty} Q(K, t_1, t_2, \delta) p(\delta) d\delta \end{aligned} \quad (18)$$

According to the definition of the correlation function, we will form its differential spectrum  $R_x(K, t_1, t_2)$  by expression:

$$R_x(K, t_1, t_2) = m_q(K, t_1, t_2, \delta) - m_x(K, t_1) * m_x(K, t_2) \quad (19)$$

Transition from the differential spectrum  $R_x(K, t_1, t_2)$  to the time domain you can perform the inverse differential transformation for two time points: at  $H_1 = t_1$  and  $H_2 = t_2$ . In result:

$$R_x(t_1, t_2) = \sum_{K=0}^{\infty} R_x(K, t_1, t_2) \quad (20)$$

Thus, the correlation function  $R_x(t_1, t_2)$  of random signal  $x(t, \delta)$  is determined by the sum of all discrete differentials

$R_x(K, t_1, t_2)$ , which can be calculated by expressions (13)-(19).

In case when  $t = t_1 = t_2$  correlation function  $R_x(t)$  is equal to the variance  $D_x(t)$ , the algorithm for calculating the correlation function coincides with the algorithm for determining the variance.

Consider a method of modeling random processes based on differential transformations of a random impulse signal, which is a signal of the hidden transmitters. The simulation will be carried out in order to obtain the signal parameters for detecting and recognizing these signals against the background of legal airwaves.

To do this, we present a random signal as a random process implementation  $X(t, \Omega)$ , which is an integral representation of the solution of a stochastic nonlinear differential equation of the form

$$\frac{dX(t, \omega)}{dt} = ae^{-\frac{1}{2}(\omega t - t_0)}, \omega \in \Omega, a = \text{const}, \quad (21)$$

with initial conditions

$$X(0, \omega) = 0 \quad (22)$$

where  $t_0$  the time of the signal appear,  $a$  - signal amplitude,  $\Omega$  - random excitation, which is a random magnitude of the space of elementary events,  $\Omega$  which is evenly spaced  $[-\alpha; \alpha]$ .

To determine the numerical parameters of a random process  $X(t, \Omega)$ , expectation  $m_x(t)$  and variance  $(D_x(t))$ , for the moment of time  $t = \tau$  we make a differential transformation for the right-hand side of equation (21) by the following rule

$$\underline{X}(k, \omega) = \frac{H^k}{k!} \left( \frac{d^k X(t, \omega)}{dt^k} \right)_{t=0} \quad (23)$$

where  $X(t, \Omega)$  - the original, which is a continuous function having the property of differentiation by  $t$  an infinite number of times, but is bounded by all its derivatives, and which is the solution of equation (23);

$\underline{X}(k, \omega)$  - differential image of the original, which is a discrete function of an integer argument  $k = 0, 1, 2, \dots$ .

$H$  - scale constant with argument dimension  $t$  and in many cases it is chosen in such a way that inequality is satisfied

$$0 \leq t \leq H \quad (24)$$

that is,  $H$  - the length of the time interval at which the characteristics of the signal representing the right-hand side of the equation are observed (21).

For equation (21), the image of the differential transform (23) has the form

$$\underline{X}(0, \omega) = 0 \quad \underline{X}(k, \omega) = \frac{H^k}{k!} (-1)^{k-1} 2^{1-k} \omega^{k-1} a e^{\frac{1}{2} t_0} \quad (25)$$

The results of the calculations for the five values are shown in Table 1.

**Table 1.** Calculation results

$k$	$X(k, \omega)$
1	$H a e^{\frac{1}{2} t_0}$
2	$-\frac{1}{4} H^2 \omega a e^{\frac{1}{2} t_0}$
3	$\frac{1}{24} H^3 \omega^2 a e^{\frac{1}{2} t_0}$
4	$-\frac{1}{192} H^4 \omega^3 a e^{\frac{1}{2} t_0}$
5	$\frac{1}{1920} H^5 \omega^4 a e^{\frac{1}{2} t_0}$

By performing the inverse transformation according to the formula

$$X(t, \Omega) = \sum_{k=0}^{\infty} \left( \left( \frac{t}{H} \right)^k \underline{X}_k(k, \omega) \right) \quad (26)$$

the solution of equation (23) with initial conditions (24), the right side of which corresponds to the image (26) is

$$X(t, \omega) = \sum_{k=1}^{\infty} \left( \left( \frac{t}{H} \right)^k \frac{H^k}{k!} (-1)^{k-1} 2^{1-k} \omega^{k-1} a e^{\frac{1}{2} t_0} \right),$$

or

$$X(t, \omega) = \sum_{k=1}^{\infty} \left( \frac{t^k}{k!} (-1)^{k-1} 2^{1-k} \omega^{k-1} a e^{\frac{1}{2} t_0} \right) \quad (27)$$

Expectation  $m_X(t)$  of random process  $X(t, \Omega)$  in the general case is determined by the formula

$$m_X(t) = E(X(t, \Omega)) = \sum_{k=1}^{\infty} \left( \left( \frac{t}{H} \right)^k E(\underline{X}(k, \omega)) \right) \quad (28)$$

If the probability density distribution of a random  $p_{\Omega}(\omega)$  variable is given  $\Omega$ , we obtain:

$$E(\underline{X}(k, \omega)) = \underline{m}_X(k) = \int_{-\infty}^{\infty} \underline{X}(k, \omega) p_{\Omega}(\omega) d\omega \quad (29)$$

Given the assumption made, the random variable is evenly distributed over the segment  $[-\alpha; \alpha]$ . Then:

$$p_{\Omega}(\omega) = \begin{cases} \frac{1}{2\alpha}, & \omega \in [-\alpha; \alpha] \\ 0, & \omega \notin [-\alpha; \alpha] \end{cases} \quad (30)$$

Using (24), (25) and (30), we have:

$$\underline{m}_X(k) = \frac{H^k}{k!} (-1)^{k-1} \frac{1}{2\alpha} 2^{1-k} a e^{\frac{1}{2} t_0} \int_{-\alpha}^{\alpha} \omega^{k-1} d\omega,$$

After the integration, we get

$$\underline{m}_X(k) = \frac{H^k}{k!} (-1)^{k-1} \frac{1}{2\alpha} 2^{1-k} \frac{\alpha^k - (-\alpha)^k}{k} a e^{\frac{1}{2} t_0}. \quad (31)$$

The results of the calculations for the five values are shown in Table 2.

**Table 2.** Calculation results

$k$	$\underline{m}_X(k)$
1	$H a e^{\frac{1}{2} t_0}$
2	0
3	$\frac{1}{18} H^3 \alpha^2 a e^{\frac{1}{2} t_0}$
4	0
5	$\frac{1}{600} H^5 \alpha^4 a e^{\frac{1}{2} t_0}$

Since at even values  $k$  the additives on the right-hand side of equality (31) are equal to 0, then, putting  $k = 2m - 1$ , in its final form, expression (31) has the form

$$\underline{m}_X(2m - 1) = H^{2m-1} \frac{\alpha^{2m-2}}{(2m-1)!(2m-1)} 2^{2-2m} a e^{\frac{1}{2} t_0}, \quad m = 1, 2, \dots \quad (32)$$

Then, using (8), we have

$$m_X(t) = \sum_{k=1}^{\infty} \left( \left( \frac{t}{H} \right)^{2m-1} H^{2m-1} \frac{\alpha^{2m-2}}{(2m-1)!(2m-1)} 2^{2-2m} a e^{\frac{1}{2} t_0} \right), \quad m = 1, 2, \dots,$$

or

$$m_X(t) = \sum_{k=1}^{\infty} t^{2m-1} \frac{\alpha^{2m-2}}{(2m-1)!(2m-1)} 2^{2-2m} a e^{\frac{1}{2} t_0}, \quad m = 1, 2, \dots \quad (33)$$

Putting  $m = 1, m = 2, m = 3$ , finally we obtain the function of expectation of a given random process from time to time:

$$m_X(t) = \left( t + \frac{\alpha^2}{72} t^3 + \frac{\alpha^4}{9600} t^5 \right) a e^{\frac{1}{2} t_0}.$$

Using the definition of the variance of a random process  $X(t, \Omega)$ , we have

$$D_X(t) = E, \quad (34)$$

By entering an additional variable  $V(t, \omega) = (X(t, \omega))^2$ , we will get

$$D_X(t) = E(V(t, \omega)) - (m_X(t))^2. \quad (35)$$

Using the differential transform (3), the dispersion image has the form

$$\underline{D}_X(k) = \int_{-\infty}^{\infty} \left( \sum_{l=0}^k \left( \underline{X}((k-l), \omega) \underline{X}(l, \omega) \right) \right) p_{\Omega}(\omega) d\omega - \sum_{l=0}^k \left( \underline{m}_X(k-l) \underline{m}_X(l) \right). \quad (36)$$

In our case, using (5) and (12), we have

$$\underline{D}_X(k) = \frac{1}{2\alpha} \int_{-\alpha}^{\alpha} \left( \sum_{l=0}^k \left( \underline{X}((k-l), \omega) \underline{X}(l, \omega) \right) \right) d\omega - \sum_{l=0}^k \left( \underline{m}_X(k-l) \underline{m}_X(l) \right)$$

$$k=1: \quad \underline{D}_X(1) = 0, \text{ так как } \underline{X}(0, \omega) = \underline{m}_X(0) = 0;$$

$$k=2: \quad \underline{D}_X(2) = \frac{1}{\alpha} \int_{-\alpha}^{\alpha} \left( \left( \underline{X}(1) \right)^2 \right) d\omega - 2 \underline{m}_X(1) \underline{m}_X(2) = 2H^2 a^2 e^{t_0};$$

$$k=3: \quad \underline{D}_X(3) = \frac{2}{\alpha} \int_{-\alpha}^{\alpha} \left( \underline{X}(1) \underline{X}(2) \right) d\omega - 2 \underline{m}_X(1) \underline{m}_X(2) = 0;$$

$$k=4:$$

$$\underline{D}_X(4) = \frac{1}{\alpha} \int_{-\alpha}^{\alpha} \left( 2 \underline{X}(1) \underline{X}(3) + \left( \underline{X}(2) \right)^2 \right) d\omega - 2 \underline{m}_X(1) \underline{m}_X(3) = \frac{1}{72} H^4 a^2 e^{t_0} \alpha^2;$$

$$k=5: \quad \underline{D}_X(5) = \frac{1}{\alpha} \int_{-\alpha}^{\alpha} \left( \underline{X}(1) \underline{X}(4) + \underline{X}(2) \underline{X}(3) \right) d\omega - \left( \underline{m}_X(0) \underline{m}_X(5) + 2 \underline{m}_X(1) \underline{m}_X(4) + 2 \underline{m}_X(2) \underline{m}_X(3) \right) = 0$$

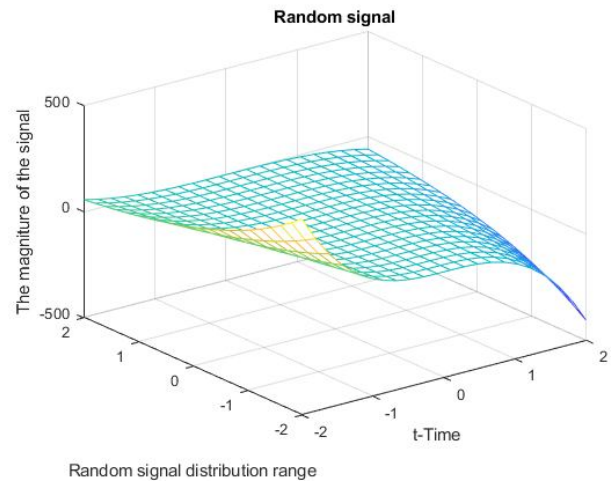
Then, according to (36):

$$D_X(t) = \sum_{k=1}^{\infty} \left( \left( \frac{t}{H} \right)^k \underline{D}(k) \right) = \left( \frac{1}{72} \alpha^2 t^4 + t^4 \right) a^2 e^{t_0}, \quad k = 1, 2, 3, 4, 5. \quad (37)$$

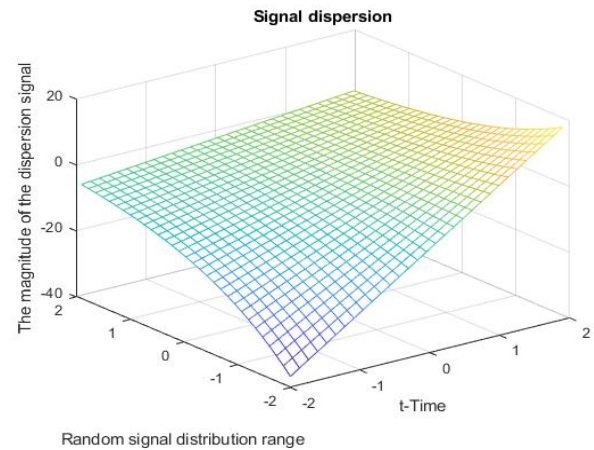
According to (37) and  $k = 1, 2, 3, 4, 5$ , we obtain the analytical dependence of the random signal  $X(t, \omega)$ , as a function of time

$$X(t, \omega) = \left( \frac{1}{1920} \omega^4 t^5 - \frac{1}{192} \omega^3 t^4 + \frac{1}{24} \omega^2 t^2 + t \right) a e^{\frac{1}{2} t_0} \quad (38)$$

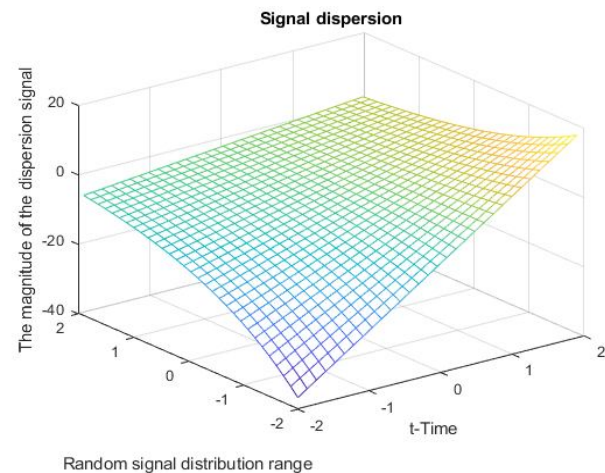
For a clear presentation of the analytical results obtained, we will perform a random signal simulation in the MathLab software environment (Figs. 1 - 3).



**Figure 1.** Random signal describing a digital signal of hidden transmitter



**Figure 2.** Expectation of a signal of a digital signal of hidden transmitter



**Figure 3.** The dispersion of the signal of a digital signal of hidden transmitter

From the above simulation results we can see that the mathematically determined expectation and variance,

determined in the calculation, completely repeats the given signal near the zero value, which is a proof of the reliability of the modeling of the process.

However, it should be noted that as the signal time or range increases, the discrepancy increases. Therefore, the above method can guarantee the calculation of a random signal with high quality, since the investigated signals are short-lived.

### 3. CONCLUSION

A math method for calculating the basic statistical parameters of random signals emitting by hidden transmitters is proposed. The proposed math method is based on a model of differential transformations of random signals. A mathematical model has been constructed that most accurately displays the signal and allows to determine the parameters of the signal.

The technique of calculating the parameters of signals of digital hidden transmitters allows to obtain the following statistical characteristics:

expectation of a random signal, consisting of the sum of all expectations of the differential spectrum, under certain initial conditions;

random signal variance, which is calculated by determining the differential spectrum and calculating the variance for each discrete.

The approach to determining the correlation function of a random signal is grounded. It is proved that it is constructed on two differential spectra and consists of the sum of all discrete differential spectra determined for the correlation function.

A random signal is simulated to determine its parameters: expectation and variance. The mathematical model allows the results to be conveniently obtained for further digital analysis. The obtained graphical simulation results confirm the correctness of the analytical results. The proposed math method allows to analyze statistical characteristics in order to detect and recognize a random signal against the background of legal radio signals.

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