



Deterministic Optimization Method (DOM) of Permanent Magnet Flux Switching Machine (PMFSM) for Application of Electric Bikes

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ABSTRACT

Permanent magnet flux switching machines (PMFSM) have been attracting revived research interests for various applications over the last decades. Those machines not only inherit most merits from the conventional permanent magnet synchronous machines but also possess a simple passive and hence robust rotor. This paper presents a single-phase PMFSM analysis of initial and optimize design for electric bike applications. The result analysis of these motors has been certified using the 2D-FEA analyses. As a result, the design of optimized is significantly lesser magnet volume and high torque compare to initial motor PMFSM design by using the “deterministic optimization method” (DOM). The optimized motor design has greater average torque performance analysis up to 16% whereas the permanent magnet weight decreased at 28%. As a result, the machine becomes an efficient lightweight motor for electric vehicles and low cost because of a reduction in PM weight.

Key words: Flux-switching machine, initial, single-tooth, permanent magnet, optimization, deterministic optimization method.

1. INTRODUCTION

As the number of motor vehicles on the world's roads increases astonishingly every year, the dependence on petroleum-based fuel increases almost uncontrollably [1]. The increased use of non-renewable fossil fuels brings with their environmental problems such as the "greenhouse effect", health problems for city deliveries, and concern over the stability of fuel supply [2], [3]. Accordingly, many countries are considering banning such vehicles, and many European cities have plans to ban combustion (petrol and diesel) vehicles by 2040 including the United Kingdom, China, France, Germany, India, Ireland, Israel, and Norway [4].

Another approach to reducing environmental pollution is by using electric vehicles (EVs) [5]–[9]. Among EVs, electric bicycles (e-bikes) are receiving more attention worldwide due to their many benefits [10]. With developing technologies, the

worldwide sales of e-bicycles are predicted to grow from nearly 32 million units in 2014 to over 40 million units in 2023 [11]. As reported by Research of Navigant[12], China is expected to be the largest market of electric bikes in the world. As shown in Figure 1, referring to the sale of about 30 million units from 2016 to 2025, China is the highest compare to the other country in the world.

Nevertheless, Electric bikes are exposed to the country's guidelines and regulations [38]. The development of an electric bike for a bike-sharing system requires that two main points of concern must be considered. First is the need of the operator to focus on a reliable, simple, efficient, sustainable but profitable system [39]. A recent survey conducted indicated that cost and frequent motor fault were among the major obstacles in using e-bicycles while low-price, light-weight, longevity, and smart features were among the major incentives in using them [40].

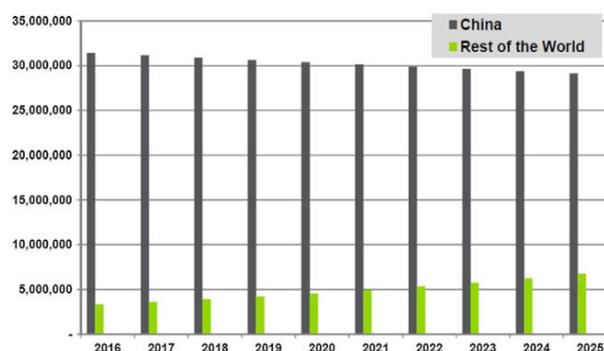


Figure 1: Annually Market Sale for Electric Bike in China and other countries in the world.

However, the motor must provide sufficient torque to overcome the road load which consists of the aerodynamic drag, the rolling resistance, the tractive force against the gravitational pull when climbing up a slope, and the tractive force for acceleration [41]. Motor for electric bikes is acceptable to have a 250 W at maximum power rating and a 25 km/hr maximum speed (wheel speed of 200 rpm) for safety reasons [42]. Moreover, the basic requirement highlighted for

electric motors used in electric bikes would be the potential of providing higher torque per motor volume development [43]. Due to space limitations, the electric motor use in an electric bike requires a short axial length and lightweight [44]. Consequently, the material selection must be a focus on economical that require low-cost material, including for production equipment and tooling, hence, the cost of processing be able to reduce.

Various researches have been dedicated to the analysis and reduction of the permanent magnet (PM). The characteristics and sensitivity analysis of design parameters influence the output torque [14]-[16]. In this paper, the design and optimization of an SRM are presented, to maximize the average torque with modify the motor geometry for optimal parameters while trying to minimize the size of the permanent magnet in the motor. The goal of this optimization was both to obtain the best possible performance of a 4slot-8pole PMFSM and to understand the most influential parameters to improve the performance. This optimization focused on the optimum number of turns as well as on the optimum shape of winding slots by keeping the winding slot area approximately constant.

The current study focuses on the comparative performance of the initial and optimization of PMFSM design for electric bike applications. Primarily, the designs are examined under no-load conditions, followed by the load analysis to determine the performance of power and torque of the motor design.

2. STRUCTURE DESIGN OF PMFSM

The primary machine geometrical dimension is identical in Figure 2. It displays the enlarged motor with the original size of the main machine parts including for example in the space between the rotor and stator, the rotor length and armature coil, and stator yoke diameter.

2.1 Design Specifications and Restriction

This model, from a practical point of view, the stator diameter for the motors has been fixed at 85 mm. The requirements and characteristics of the design of PMFSM are presented in Table I. The cross-section of the multi-tooth, single-tooth and segmental-tooth design has been shown in Figure 2. Eventually, the design was formed by using simulation software, JMAG Designer version 14.3.

Design specifications for dimensions of the motors and the workflow for 2D analysis are illustrated in Figure 3. The design specification and conditions for all the design motors should be similar. The project implementation has been divided into two-phase which are the design and analyze the performance of PMFSM respectively. Geometry editor is applied to design all the motor parts separately for example in the rotor, stator, and armature coil whereas the simulation and set of conditions of the motor formed by utilizing the JMAG-Designer.

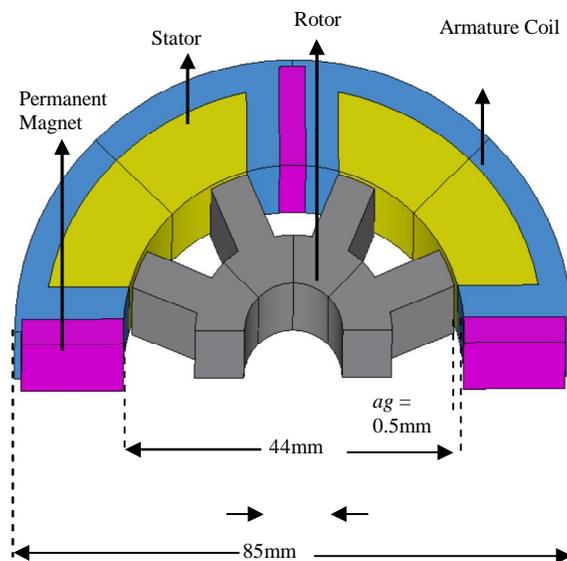


Figure 2: Main Machine Dimension of the Proposed PMFSM

Table 1: PMFSM for Electric Bike Design Requirement

Items	Parameter
Number of rotors	8
Number of stators	4
No. of phase	1
Diameter (mm)	85
inner stator radius (mm)	26.0
stator outer radius (mm)	42.5
rotor outer Radius (mm)	25.5
Rotor width (mm)	10.5
Rotor shaft (mm)	7.5
Motor stack length (mm)	30
Air gap length (mm)	0.5

2.2 Possible Combinations of PMFSM Design Parameter Sensitivity

To achieve the highest performance design, the best possible combination of the number of poles needs to be set [21]. Therefore, some relevant parameters are considered for the potential combination development of stator and rotor poles [20]. This slot and poles combination can be identified using (1) and (2).

$$N_p = K_m (K = 2, 4, 6 \dots) \quad (1)$$

$$N_r = N_s (2m - 1) + 1 \tag{2}$$

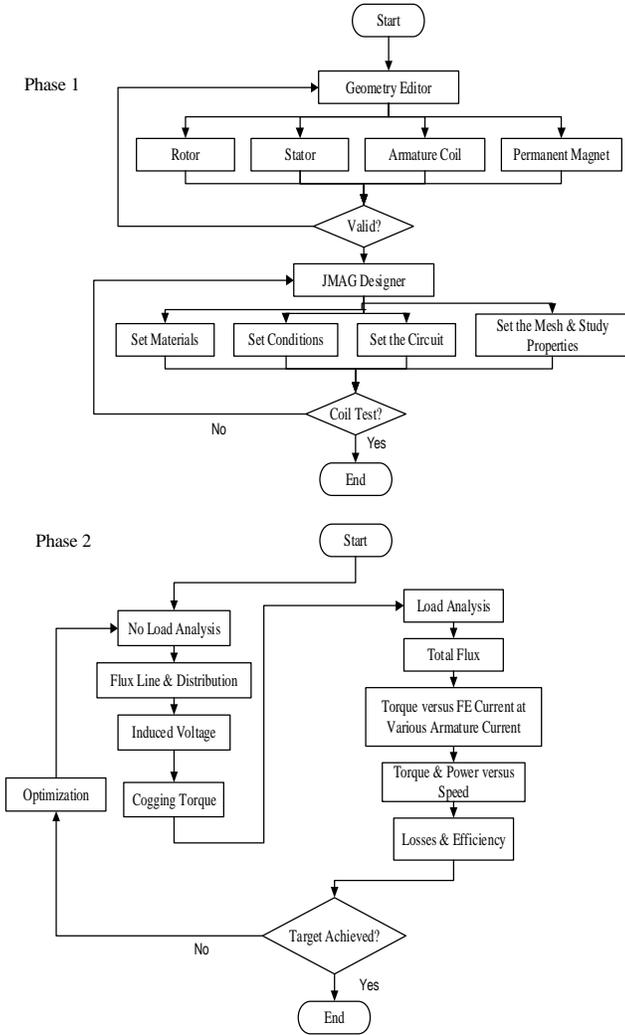


Figure 3: Analyze Performance Based on JMAG Designer with 2D FEA Analysis

The subscripts N_s and N_r will be the number of stator and rotor poles, respectively, while the number of phases is m . More generally, the number of rotor pole and slot pole especially for single-phase must be in even numbers. Therefore, the PMFSM motor can be designed as a phase winding comprises a coil that has a phase shift of 180 electrical degrees among the coils.

Preliminary, according to the design of single-tooth PMFSM, the total of stator slots area must be even numbers. Accordingly, referring to equation (3), the correlation between the stator slot and rotor pole number is N_s and N_r respectively [6].

$$N_r = N_s \left(1 \pm \frac{k_f}{2q} \right) \tag{3}$$

Therefore, the symbol of (k) and q is a natural entity with its value of 1, 2, and 3 and by the number of phases respectively. Furthermore, the electric frequency (f_e), of a proposed motor can be designated as (4).

$$f_e = N_r \cdot f_m \tag{4}$$

Consequently, f_e and f_m is the electric frequency and mechanical rotation frequency. The current of armature coil (I_a) and the number of turns (N_a) are determined through the (5) and (6), separately whereas considering diameter specification used for the copper is 0.56mm.

$$I_a = \sqrt{N_r \cdot \alpha \cdot S_a} / N_a \tag{5}$$

$$N_a = \alpha \cdot S_a / \left(\frac{A_{copper}}{2} \right)^2 \tag{6}$$

Subscripts ‘ α ’ indicates armature coil, J_a , S_a , and A_{copper} are current density, slot area (mm^2), and area of copper, respectively. Through the analysis, 240V DC bus voltage is the appropriate electrical limit for the inverter and 3.5 Arms/ mm^2 for the maximum current of the inverter, while J_a 10 Arms/ mm^2 are set to correspond to the current density in the armature coil. Since the design is focused to compare the analysis of the motor, some of the parameters remain constant. The motor stack length is set to 30mm and the air gap is 0.5mm. To ensure the flux flows uniformly without any leakage from the stator to the rotor, the motor’s filling factor (α) is set to 0.5. The power is found using equation (7), and that is proportional to the torque and speed analysis.

$$P = \frac{2\pi}{60} (T \cdot S) \tag{7}$$

3. PERFORMANCE ANALYSIS OF PMFSM OPTIMIZATION DESIGN

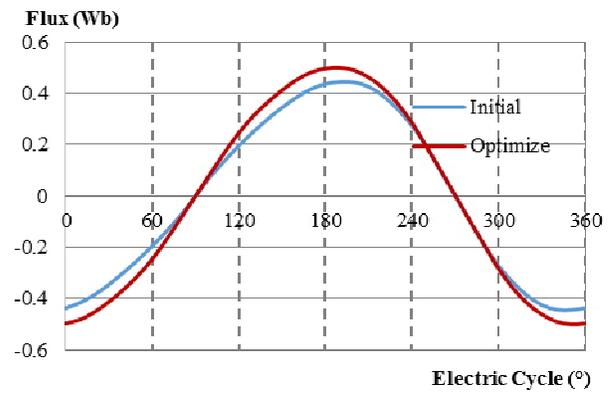
The motor of 4slot-8pole PMFSM is compared among the initial and optimize the design for electric bike application and presented in Table 2. The design analysis for comparison of the initial and final design is conducted under no load and n load analysis. The load analysis examination contains the analysis of flux linkage, cogging torque, and back electromagnetic force (back-EMF), nevertheless, at load analysis, output power and torque performance will be analyzed completely at various armature current densities (J_a).

Table 2: Initial and Optimization Key Design Criterion

Items	Initial	Optimize
Number of rotors	8	8
Number of stators	4	4
No. of phase	1	1
Number of turns	438	459
Armature slot area (mm ²)	201.6	216.8
Diameter (mm)	85	85
Stator inner radius(mm)	26.0	27.0
Stator outer radius(mm)	42.5	42.5
Rotor outer radius(mm)	25.5	26.5
Rotor inner radius(mm)	7.5	7.5
Rotor pole height	10.5	9.5
Rotor pole width (mm)	6.0	7.0
PM width (mm)	5.0	4.0
PM length (mm)	15.5	15.5
Stator back length (mm)	4.5	4.0
Rotor yoke length (mm)	7.5	9.5
Air gap length (mm)	0.5	0.5
Stator bridge(mm)	1.0	-
PM Volume (g)	70g	56g

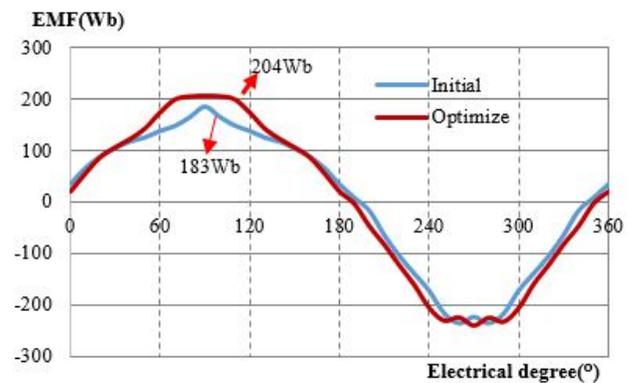
3.1 Characteristics of Magnetic Flux Linkage

The magnetic flux for both designs of initial and optimize in PMFSM is produced by magnetic flux linkage (induce voltage) and achieved by rotation of rotor at the speed of 500 r/min whereas the current density for armature is fixed at 0 Arms/mm². As can be seen in Figure 4, The magnitude rose to 0.5Wb compare to the initial design at 0.4Wb. The flux is increased due to the diminished size of the rotor pole height. Therefore, the flux magnetic may take for a short period to perform one complete cycle and minimized the flux leakage, and flux cancellation has been decreased which also influences by the magnitude of flux linkage.

**Figure 4:** Initial and optimize flux linkage performance comparison

3.2 Electromagnetic Force (Back-EMF)

The supplementary analysis of the initial and optimized topology of PMFSM on back-electromagnetic force (EMF) at an open circuit is conducted at the speed of 500 r/min. The achieved results of back-EMF are plotted in Figure 5. The voltages generated by optimized design are greater compare to the initial design since back-EMF is directly related to the magnetic flux linkage. Since the magnitude of magnetic flux is increased so back-EMF is also enlarged. Moreover, higher back-EMF of optimized design can be employed for a regenerative braking system to charge the source [13].

**Figure 5:** Back electromagnetic Force at a speed of 500 r/min

3.3 Performance for Analysis of Load Performance

After all, by setting the armature current density at maximum condition, the output torque and power at various armature current density (J_a) are set to 0 to 10 A_{rms}/mm² illustrated in Figure 6 and Figure 7. In the preliminary design, at the base speed of 500 r/min, the performance of output torque is 4.1Nm whereas corresponding power is 166W as seen in Figure 6. Using the application of the design optimization method (DOM), the performance of output torque at load analysis has increased significantly at 5 Nm.

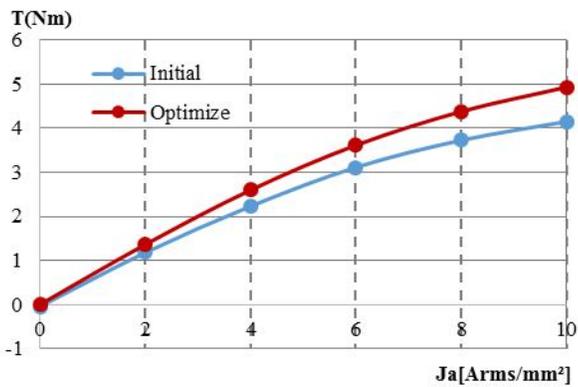


Figure 6: Analysis of Torque at various Ja

The improvement of torque performance initially resulting from the enlargement in the area of armature current (AC), notably accommodates a higher number of turns and design structure of permanent magnet. Moreover, the PM weight downsizing and the optimal split ratio selection also contributing to the development of torque performance. Accordingly, the 198W of output power is generated by an optimized design compared to the initial design of 166W. Besides, after using the design optimization method (DOM), 18% of the output torque is increased slightly, greater than the performance of the initial design. This condition revealed that the high value of flux linkage will develop greater torque performance. It remains, the overall effect on the proposed designs is pictured in Table 3.

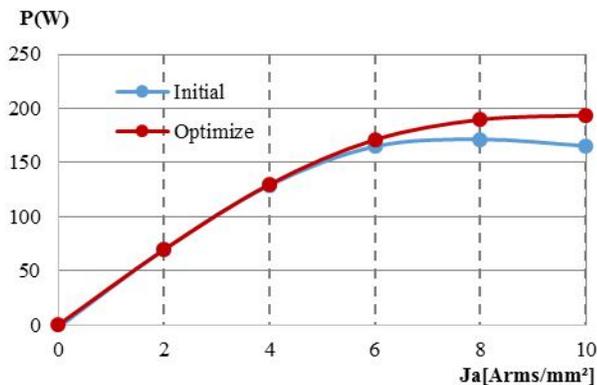


Figure 7: Analysis of Power at various Ja

Table 3: Comparison Performance of Initial and Optimization for PMFSM Design

Number of Slot Pole	Initial	Optimize
Flux (Wb)	0.4	0.5
Cogging torque (Nm)	5.20	5.5
B.EMF	183.0	204.0
Maximum torque (Nm)	4.1	5.0
Maximum power (W)	165.5	193.7

4. CONCLUSION

The present discussion has demonstrated the comparison performance of initial and optimize the design of single-phase 4slot-8pole PMFSM design for electric bike applications. The comparison was made under load and no-load condition that contains few analyses. Successively, the optimized motor has greater performance compared to initial design which reduced permanent magnet weight and better torque performance. The optimized PMFSM motor design has increased the performance of output torque to 18% whereas, at the same time, the weight of permanent magnet is decreased by up to 20%. Therefore, according to a lessening in PM weight, the motor is lightweight and potentially to reduce the cost of the motor. In the final analysis, it can be summarized with all the analytical surveys that the optimized design of PMFSM with 4slot-8pole is one of the influential design motors for electric bike applications. It is foreseeable that more interesting researches would be devoted to the PMFS machines, especially on the aspects of less PM, easy and inexpensive manufacturing, and further optimization.

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