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Numerical simulation of the acoustic wave equation by the method of reverse time migration

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ABSTRACT

The paper deals with seismic method of studying the structure of the earth crust namely reverse time migration (RTM) [1]. Migration is described by a conventional wave equation and is performed by propagating the data down through the acoustic pressure to the ground and it is constrained where the structure and acoustic pressure field generate more complex arrivals, such as turn waves and prisms. Since the wave pressure is variable in the earth interior, this wave equation has different coefficients [2]. Also, this paper discusses the existence and uniqueness of solutions of partial differential equations using Fourier transform [3]. Linearization is used for finding approximate solutions, which is direct problem. This model is used in the migration method to construct an image of reflecting boundaries. In addition, in this paper we study the solution of inverse problems of the wave equation allowing the study of the characteristics of the earth underground medium.

Key words: Acoustic wave equation, full waveform inversion, Fourier transformation,reverse time migration.

1. INTRODUCTION

We consider the acoustic approximation described by the wave equation:

$$\frac{1}{C^2(x)}\frac{\partial^2 P(x,t)}{\partial t^2} = \frac{\partial^2 P}{\partial x^2} + f(x,t)\cdot\delta(x-x_0) \quad (1)$$

where, the wave speed $C(x)$ is seen as a function of position;

$$C(x) = \begin{cases} C_0(x), P \le x \le H \\ C_0(H), x > H \end{cases} \quad (2)$$

$f(x,t)$ is a forcing term **Error! Reference source not found.**.

The function is defined in the next interval: $0 \le x \le \infty$, $t \ge 0$ with zero initial conditions:

$$P(x,0)\big|_{t=0} = \frac{\partial P(x,0)}{\partial t}\bigg|_{t=0} = 0 \quad (3)$$

And boundary conditions [4]:

$$P(0,t)\big|_{x=0} = 0,$$

$$\left(\frac{\partial P(x,t)}{\partial t} + C\frac{\partial P(x,t)}{\partial x}\right)\bigg|_{x=H} = 0 \quad (4)$$

Replace the discrete with the continuous $P(x,t)dt$. Then change the sum to an integral, and the equation (1) become

$$\frac{d^2\hat{P}(x,\omega)}{dx^2} + \frac{1}{C^2(x)}\omega^2\hat{P}(x,\omega) + $$
$$+ F(x,\omega)\cdot\delta(x-x_0) = 0 \quad (5)$$

To find complete solution of (1) we use inverse Fourier transformation for the solution (5) [5]:

$$P(x,t) := \frac{1}{\sqrt{2\pi}}\int_R \left(\frac{iC_0}{2\omega}\cdot e^{\frac{i\omega}{C_0}|-x_0|}\cdot e^{\frac{i\omega}{C_0}x} - \right.$$
$$\left. - \frac{iC_0}{2\omega}\cdot e^{\frac{i\omega}{C_0}|x-x_0|}\right)e^{i\omega t}d\omega. \quad (6)$$

The simplicity of the sequential solution algorithm is believed to be a new contribution as well.

2. METHOD OF RESEARCH

2.1 Numerical implementation

In order to solve the inverse problem, we derived an auxiliary problem from the basic mathematical model.

Auxiliary task:

$$\begin{cases} \dfrac{\partial\Delta\sigma}{\partial t} = \dfrac{\partial^2\Delta P}{\partial x^2} \\ \Delta P\big|_{t=0} = 0, \dfrac{\partial\Delta P}{\partial t}\bigg|_{t=0} = 0 \\ \Delta P\big|_{x=0} = 0 \\ \Delta(C\cdot\sigma)\big|_{x=H} + \dfrac{\partial\Delta P}{\partial x}\bigg|_{x=H} = 0, \end{cases} \quad (7)$$

where $\sigma = \dfrac{1}{C^2}\cdot\dfrac{\partial P}{\partial t}$.

Then we derive the conjugate problem using an arbitrary function ψ. Conjugated problem.

$$\begin{cases} \frac{\partial}{\partial t}(\frac{1}{C^2}\frac{\partial \psi}{\partial t}) - \frac{\partial^2 \psi}{\partial x^2} = 0 \\ \psi\big|_{t=t_{\max}} = 0, \frac{\partial \psi}{\partial t}\Big|_{t=t_{\max}} = 0 \\ \psi\big|_{x=0} = 0 \\ C \cdot \psi\big|_{x=H} + \frac{\partial \psi}{\partial x}\Big|_{x=H} = 0 \end{cases} \tag{8}$$

Using the method of finite difference schemes, we solve our equation **Error! Reference source not found.**:

$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \frac{dx}{C^{2(x)}} \int_{t_{j-1}}^{t_j} \frac{\partial^2 P(x,t)}{\partial t^2} dt = = \int_{t_{j-1}}^{t_j} \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \frac{\partial^2 P(x,t)}{\partial x^2} dxdt + A, \tag{9}$$

where

$$A = \int_{t_{j-1}}^{t_j} \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x,t) \cdot \delta(x - x_0) dx .$$

Then,

$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \frac{dx}{C^{2(x)}} \left[\frac{\partial P(x_i,t_j)}{\partial t} - \frac{\partial P(x_i,t_{j-1})}{\partial t} \right] dx = = \int_{t_{j-1}}^{t_j} \left[\frac{\partial P(x_{i+1/2},t_j)}{\partial x} - \frac{\partial P(x_{i-1/2},t_j)}{\partial x} \right] dt + A, \tag{10}$$

using interpolation, we get

$$\frac{\Delta x}{C^2(x_i)} \left[\frac{P(x_i,t_{j+1}) - P(x_i,t_j)}{\Delta t} - \frac{P(x_i,t_j) - P(x_i,t_{j-1})}{\Delta t} \right] \approx \approx \left[\frac{P(x_{i+1},t_j) - P(x_i,t_j)}{\Delta x} - \frac{P(x_i,t_j) - P(x_{i-1},t_j)}{\Delta x} \right] \Delta t + + \int_{t_{j-1}}^{t_j} dt \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x,t) \cdot \delta(x-a) dx. \tag{11}$$

And, our model takes the form

$$\begin{cases} \frac{1}{C^2(x_i)} \cdot \frac{y_i^{j+1} - 2y_i^{\,j} + y_i^{\,j-1}}{\Delta t^2} = \\ = \frac{y_{i+1}^{\,j} - 2y_i^{\,j} + y_{i-1}^{\,j}}{\Delta x^2} + \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x,t) \cdot \delta(x-a) dx, \\ \left(i = 1,2,...N-1, \quad j = 1,2,...M-1\right), \\ y_i^{\,0} = 0, \quad \left(i = 0,1,...N\right), \\ \frac{y_i^{\,1} - y_i^{\,0}}{\Delta t} = 0, \quad \left(i = 0,1,...N\right), \\ y_0^{\,j} = 0, \quad \left(j = 1,2,...M-1\right), \\ \frac{y_N^{\,j+1} - y_N^{\,j}}{\Delta t} + C\frac{y_N^{\,j} - y_{N-1}^{\,j}}{\Delta x} = 0, \\ \left(j = 1,2,...M-1\right), \end{cases} \tag{12}$$

when

$$\int_{-\infty}^{+\infty} \delta(x - x_0) dx = 1$$

and

$$\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x_i,t_j) \cdot \delta(x-a) dx = \begin{cases} f(a,t_j), i = N_0 \\ 0, i \neq N_0 \end{cases} \tag{13}$$

We can rewrite the equation to calculate the wave field at time step n + 1 from the wave field at the previous time steps $\left(i = 1,2,...N-1,\ j = 1,2,...M-1\right)$:

$$y_i^{\,j+1} = 2y_i^{\,j} - y_i^{\,j-1} + \frac{C^2\Delta t^2}{\Delta x^2}(y_{i+1}^{\,j} - 2y_i^{\,j} + +y_{i-1}^{\,j}) + C^2\Delta t^2 \cdot \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x,t) \cdot \delta(x-a) dx, \tag{14}$$

If we use c on Conjugate model, it will be:

$$\begin{cases} \frac{1}{C^2} \cdot \frac{u_i^{\,j+1} - 2u_i^{\,j} + u_i^{\,j-1}}{\Delta t^2} - \frac{u_{i+1}^{\,j} - 2u_i^{\,j} + u_{i-1}^{\,j}}{\Delta x^2} = 0, \\ (i = 1,2,...N-1, \quad j = 1,2,...M-1) \\ u_i^{\,M} = 0, i = 0,1,...N \\ \frac{u_i^{\,M} - u_i^{\,M-1}}{\Delta t} = 0, i = 0,1,...N \\ u_0^{\,j} = 0,\ j = 1,2,...M-1 \\ Cu_N^{\,j} + \frac{u_N^{\,j} - u_{N-1}^{\,j}}{\Delta x} = 0,\ j = 1,2,...M-1 \end{cases} \tag{15}$$

The construction of the image is in three stages. First, the wave field from each source is modeled, then the field from seismograms launched in the reverse time and taken as sources is modeled. Then the correlation of the two obtained fields and the subsequent summation for all sources is performed. The resulting field is the image of the environment. Among the different seismic migration methods for different situations, RTM is the only method that is able to use all types of seismic waves that can be calculated numerically [**Error! Reference source not found.**]. The procedure of migration in the reversed time is applied separately to each seismogram of the specified source position x_s . For it, a partial image is calculated by integrating the product of two wave fields over time:

$$g(x,x_s)=\int_0^T P(x,t)P_{sr}(x,t)dt, \tag{16}$$

where $P(x,t)$ is a straight field from a point source located at a point x_s with the impulse $f(x,t)$, i.e. the solution of the wave equation (for high speed models $C(x)$) with zero initial data (Cauchy problem); $P_{sr}(x,t)$ is a reversed field generated by the data at the receiving points and continued in the reverse time:

$$\frac{1}{C^2(x)}\frac{\partial^2 P(x,t)}{\partial \zeta^2}=\sum_r \delta(x-x_0)d_r(x_s,T-\zeta)\cdot \tag{17}$$

with zero initial **Error! Reference source not found.** at $\zeta=0$.

Using the finite difference schemes **Error! Reference source not found.**, we calculate the wavefield at time step n -1 from the wavefield:

$$y_i^{j-1}=2y_i^{\,j}-y_i^{\,j-1}+\frac{C^2\Delta t^2}{\Delta x^2}(y_{i+1}^{\,j}-2y_i^{\,j}+$$

$$+y_{i-1}^{\,j})+C^2\Delta t^2\cdot\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x,t)\cdot\delta(x-a)dx, \tag{18}$$

$$i=1,2,...N-1,\ j=M-1,M-2,...2$$

Summation in the right part **Error! Reference source not found.** is conducted on all receivers. The function on the right side $d_r(x_s,T-\zeta)$ is the data recorded by the receivers at the points x_r; T maximum observation time.

To get the full image, you need to add all the partial images (for different positions of the sources):

$$G(x)=\sum_s g(x,x_s), \tag{19}$$

where $G(x)$ is a complete image of the reflecting boundaries, which is then used in geological interpretation.

2.2 Approximation and convergence

Real-valued function f is differentiable at the point a then it has a linear approximation at the point a. This means that there exists a function h_1 such that

$$f(x)=f(a)+f'(a)(x-a)+h_1(x)(x-a),$$

$$\lim_{x\to a}h_1(x)=0 \tag{20}$$

Approximation

$$y_i^{j+1}=y\left(x_i,t_j+\Delta t\right)=y\left(x_i,t_j\right)+y'\left(x_i,t_j\right)\Delta t+$$

$$+y''\left(x_i,t_j\right)\frac{\Delta t^2}{2!}+y'''\left(x_i,t_j\right)\frac{\Delta t^3}{3!}+O\left(\Delta t^4\right) \tag{21}$$

$$y_i^{j-1}=y\left(x_i,t_j-\Delta t\right)=y\left(x_i,t_j\right)-y'\left(x_i,t_j\right)\Delta t+$$

$$+y''\left(x_i,t_j\right)\frac{\Delta t^2}{2!}-y'''\left(x_i,t_j\right)\frac{\Delta t^3}{3!}+O\left(\Delta t^4\right) \tag{22}$$

$$\frac{y_i^{\,j+1}-2y_i^{\,j}+y_i^{\,j-1}}{\Delta t^2}\approx y''(x_i,t_i)+O(\Delta t^2) \tag{23}$$

$$\frac{y_{i+1}^{\,j}-2y_i^{\,j}+y_{i-1}^{\,j}}{\Delta t^2}\approx y''(x_i,t_i)+O(\Delta x^2) \tag{24}$$

The approximation is equal to $O(\Delta x^2,\Delta t^2)$.

Convergence

$$\varepsilon_i^{\,j+1}=y_i^{\,j+1}-u_i^{\,j+1}\Rightarrow y_i^{\,j+1}=\varepsilon_i^{\,j+1}+u_i^{\,j+1} \tag{25}$$

$$\frac{1}{C^2(x_i)}\frac{\varepsilon_i^{\,j+1}+u_i^{\,j+1}-2\left(\varepsilon_i^{\,j}+u_i^{\,j}\right)+\varepsilon_i^{\,j-1}+u_i^{\,j-1}}{(\Delta t)^2}=$$

$$=\frac{\varepsilon_{i+1}^{\,j}+u_{i+1}^{\,j}-2\left(\varepsilon_i^{\,j}+u_i^{\,j}\right)+\varepsilon_{i-1}^{\,j}+u_{i-1}^{\,1}}{(\Delta x)^2}+$$

$$+\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x_i,t_j)\cdot\delta(x-a)dx \tag{26}$$

$$\frac{1}{C^2(x_i)}\frac{\varepsilon_i^{\,j+1}-2\varepsilon_i^{\,j}+\varepsilon_i^{\,j-1}}{(\Delta t)^2}-\frac{\varepsilon_{i+1}^{\,j}-2\varepsilon_i^{\,j}+\varepsilon_{i-1}^{\,j}}{(\Delta x)^2}=$$

$$=-\frac{1}{C^2(x_i)}\frac{u_i^{\,j+1}-2u_i^{\,j}+u_i^{\,j-1}}{(\Delta t)^2}+\frac{u_{i+1}^{\,j}-2u_i^{\,j}+u_{i-1}^{\,j}}{(\Delta x)^2}+$$

$$+\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x_i,t_j)\cdot\delta(x-a)dx \tag{27}$$

$$y_i^{\,j}=e^{at}\cdot e^{I\lambda_m x_i},\quad I^2=-1, \tag{28}$$

$$y_{i+1}^{\,j}=e^{at}e^{I\lambda_m(x_i+\Delta x)}=e^{at}e^{I\lambda_m(i+1)\Delta x},$$

$$y_{i-1}^{\,j}=e^{at}e^{I\lambda_m(x_i-\Delta x)}=e^{at}e^{I\lambda_m(i-1)\Delta x} \tag{29}$$

$$y_i^{\,j+1}=e^{a(t+\Delta t)}e^{I\lambda_m x_i},\quad y_i^{\,j-1}=e^{a(t-\Delta t)}e^{I\lambda_m x_i}, \tag{30}$$

Suppose that

$$S=\frac{C^2(x_i)}{(\Delta x)^2}\cdot(\Delta t)^2,\quad G=C^2(x_i)\cdot(\Delta t)^2,\quad K=\frac{y_i^{\,j+1}}{y_i^{\,j}} \tag{31}$$

$$y_i^{\,j+1}=y_i^{\,j}[2-2S]+S[y_{i-1}^{\,j}+y_{i+1}^{\,j}]-y_i^{\,j-1}+$$
$$+G\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x_i,t_j)\cdot\delta(x-a)dx \tag{32}$$

$$\frac{y_i^{\,j+1}}{y_i^{\,j}}=\left[2-2S\right]+[e^{I\lambda_m\Delta x}+e^{-I\lambda_m\Delta x}]\cdot S-e^{-a\Delta t}+$$
$$+G\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x_i,t_j)\cdot\delta(x-a)dx=\left[2-2S\right]+2\cos\lambda_m\Delta x\cdot S-$$
$$-\cos\Delta t+a\sin\Delta t+G\int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} f(x_i,t_j)\cdot\delta(x-a)dx \tag{33}$$

$$|K|=\sqrt{R_e^{\,2}+I_m^{\,2}} \tag{34}$$

We consider the numerical implementation of finite-difference computations for simplicity in the two-dimensional case [6-11].and which illustrates the exact solution, forward modeling and migration [12-17]. Let the model be given on a regular rectangular grid with nodes (i,j). Applying standard algorithms for the numerical solution, get the time value of the mileage at each point (i,j). Then, the solution of equation in is calculated by the finite-difference method using an explicit scheme on shifted grids at each time step.

Since the wave velocity is variable in the earth interior, we use two-layer system. The high-speed model is shown in color, diverging form the source Figure 1.

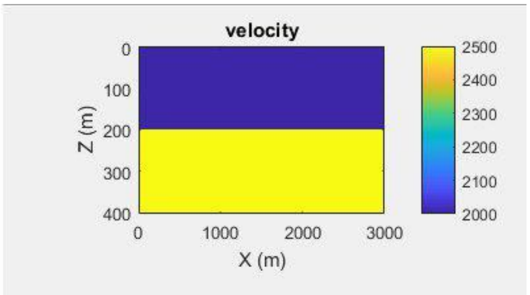

Figure 1: The high-speed model

Step of the spatial grid (*dx*) was taken to be 5 m. The source used a Riker pulse with a dominant frequency of 25 Hz. In each of the calculations, the source was located in the center of the region with time step 0.001 s.

Firstly, lets construct exact solution using formula **Error! Reference source not found.** with

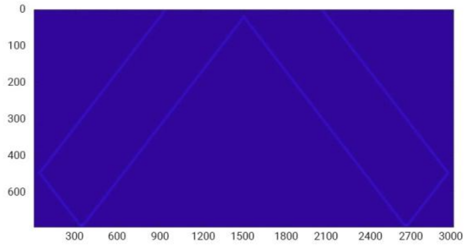

Figure Error! No text of specified style in document.**:** Exact solution

At the second stage there is a numerical solution of the wave equation **Error! Reference source not found.**.

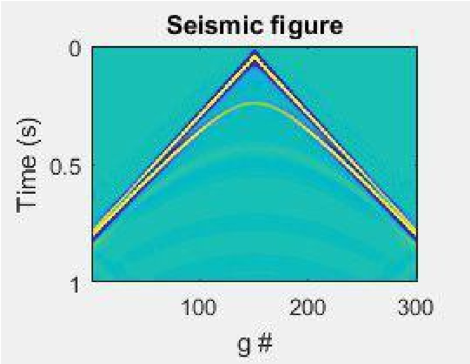

Figure 2: Seismic gather

It can be seen that in the high-speed layer the width increases, as it corresponds to the duration of the signal. For example, in the article, the source used a Riker wavelet **Error! Reference source not found.** with a dominant frequency of 25 Hz.

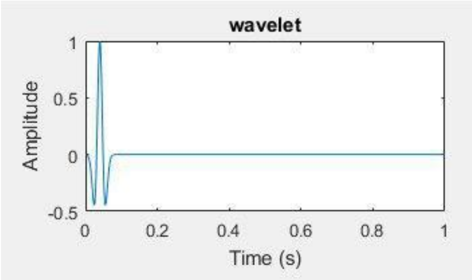

Figure 3: Ricker wavelet

To construct the image, we use **Error! Reference source not found.** and the resulting image is shown in figure 5.

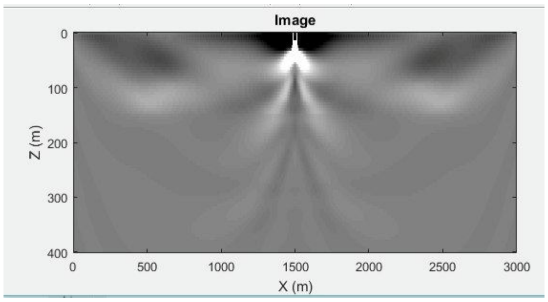

Figure 4: RTM Image with Ricker wavelet

3. CONCLUSION

In this work, we have constructed and studied a model of acoustic wave propagating in a certain region of space obtained from the solution of wave equation by Fourier transform. After that, we linearized the wave equation and obtained a direct modeling formula, which leads to a reverse time migration formula. The proposed approach is simple to implement and it avoids multiple calculations and predictions. This is important for a general overview of the general position of the earth crust.

The practical significance of our study is that the migration results show a better resolution and a more accurate position of the boundaries, which is important for the subsequent geological interpretation of the migrated seismic sections. The results obtained in this work can be used for research in the following areas:
- application of the obtained models in more complex problems, with layered structure of the medium and cracks of different shapes;
- application of the obtained models to solve specific problems seismic exploration;
- numerical experiments for the solution of model problems by the method or reverse time migration;
- comparison of the results of calculations with the results of real experiments;
- comparison of the results of calculations with the results of other numerical implementations of modeling these processes.

The procedures of direct modeling and migration were then illustrated using MATLAB code.

The continued study of the application of the inverse time migration method to the direct and inverse problems of seismic exploration and in other areas may further solve even problems of seismic exploration.

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