



Forecasting with Improved Model of Fuzzy Time Series Based on Hedge Algebras

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ABSTRACT

The aim of this paper is to introduce a new model of fuzzy time series (FTS) based on hedge algebras (HA) with three improvements compared with previous ones. Firstly, the new way of partitioning the universe of discourse is applied, secondly, the relationship groups following time points to determine relationships among fuzzy time series values are used, thirdly, weights following time points are employed to build formula for computing forecasting values. The empirical results indicate that the proposed model outperform the existing models.

Key words: Forecasting, fuzzy time series, hedge algebras, intervals.

1. INTRODUCTION

Time series forecasting continues to be a hot topic [1-3]. There are many approaches to solving this problem, in which using the fuzzy time series model is a fairly effective way.

The fuzzy time series is firstly introduced in [4], since then, there are many papers that focus on applying the model of fuzzy time series for forecasting time series. In order to modelize fuzzy time series for this task, fuzzy sets are used to quantify the linguistic terms that are the values of the fuzzy time series.

Beside fuzzy sets, hedge algebras is also considered as a tool for quantifying the linguistic terms. Paper [5] is the first study that applies hedge algebras, while paper [6] is the first one that introduces a fuzzy time series model based on hedge algebras for forecasting time series. Later study is improved in the paper [7].

In the case of using fuzzy sets for quantifying the linguistic terms, according to [8], the procedure used to apply the model of fuzzy time series for forecasting time series includes three phrases.

Phrase 1, converting the time series need forecasting, called $c(t)$, into fuzzy time series $f(t)$ by means of replacing each value of former one by a term belonging to the set of values of later one. After that, the intervals on the universe of discourse, U , of $c(t)$ are determined, finally, the fuzzy sets, which are determined based on the intervals, are used to quantify the values of $f(t)$.

Phrase 2, setting relationships among values of $f(t)$.

Phrase 3, building defuzzification formula to compute forecasting values.

The number of intervals in Phrase 1 is equal to the number of values of $f(t)$. According to [9-13], the length of intervals strongly effect to forecasting results, so there are many studies that focus on optimizing the length of the intervals by different approaches.

Model of fuzzy time series based on hedge algebras used to forecast time series is also built through three phrases as mentioned above. However, at Phrase 1, hedge algebra is used to generate linguistic terms. Next, the fuzziness intervals of the terms are taken for acting as the intervals on U .

Paper [6] suggest the way using any hedge algebras to generate the linguistic terms, but it is rather difficult to do. Paper [7] introduce a hedge algebras with only two hedges, negative hedge and positive hedge, for generating the linguistic terms with an easier way to do. The paper also proposes using variations to improve forecasting quality. Accordingly, the time series need forecasting is converted into a variations, the paper's model is applied for forecasting the variations instead of the time series. Because variations of a time series that need forecasting not only contain all information, but also exactly reflect fluctuation of the time series, the paper received rather good forecasting results.

This paper suggest three improvements compared to paper [7], specifically, at Phrase 1, it applies the better different way of determining the linguistic terms, at Phrase 2, refer [14], it applies group of relationship following time to build relationships between among values of $f(t)$, and at Phrase 3, it use defuzzification formula in Yu's work [15] to determine forecasting values.

The rest of this paper is organized as follows, section 2 introduces the basis concepts of fuzzy time series and hedge algebras, section 3, the main of this paper, presents the proposed model, section 4 presents empirical results when applying the proposed model for forecasting three time series, and section 5, the last section, presents some conclusions.

2. FUZZY TIME SERIES AND HEDGE ALGEBRAS

2.1. Fuzzy time series

FTS is a set of terms, which are observed following time points, of a random variable. Some basis concepts of FTS are recalled from papers [1] in the following.

Definition 1. Let $Y(t)$ ($t = \dots, 0, 1, 2, \dots$), a subset of R^l , be the universe of discourse on which $f_i(t)$ ($i = 1, 2, \dots$) are defined and $F(t)$ is the collection of $f_i(t)$ ($i = 1, 2, \dots$). Then $F(t)$ is called FTS on $Y(t)$ ($t = \dots, 0, 1, 2, \dots$).

Definition 2. If for any $f_j(t) \in F(t)$, there exists an $f_i(t - 1) \in F(t - 1)$ such that there exists a fuzzy relation $R_{ij}(t, t - 1)$ and $f_j(t) = f_i(t - 1) \circ R_{ij}(t, t - 1)$ in which 'o' is the max-min composition operation, then $F(t)$ is said to be caused by $F(t - 1)$ only. Denote this as

$$f_i(t - 1) \rightarrow f_j(t)$$

or equivalently

$$F(t - 1) \rightarrow F(t)$$

If $F(t)$ is caused by $F(t-1)$ or $F(t-2)$, ..., or $F(t-m)$, then $F(t)$ is called first order. This paper use the model for forecasting time series.

2.2. Hedge Algebras

Some basic HA concepts referred from [16] are introduced in the following.

The HA is defined by means of $AX = (X, G, C, H, \leq)$, where X is set of terms, $G = \{c^+, c^-\}$ is the collection of primary generators, in which c^+ and c^- are, respectively, the negative primary and positive term belong to X , $C = \{0, 1, W\}$ is a set of constants in X , H is the set of hedges, " \leq " is a semantically ordering relation on X .

For each $x \in X$, $H(x)$ is the set of terms $u \in X$, generated from x by applying the hedges of H . $H = H^+ \cup H^-$, where $H^+ = \{h_1 < h_2 < \dots < h_p\}$, $H^- = \{h_1 < h_2 < \dots < h_q\}$ is, respectively, the set of positive and negative hedges of X . The positive hedges increase semantic tendency and vice versa with negative hedges.

If X and H are linearly ordered sets, then $AX = (X, G, C, H, \leq)$ is called *linear hedge algebras*, furthermore, if AX is added two operations Σ and Φ that are, respectively, infimum and supremum of $H(x)$, then AX is called *complete linear hedge algebras* (ClinHA).

Definition 1. Let $AX = (X, G, C, H, \leq)$ be a ClinHA. An $fm: X \rightarrow [0, 1]$ is said to be a fuzziness measure of terms in X if:

(1). $fm(c^-) + fm(c^+) = 1$ and $\sum_{h \in H} fm(hu) = fm(u)$, for $\forall u \in X$;

(2). For the constants $0, W$ and 1 , $fm(0) = fm(W) = fm(1) = 0$;

(3). For $\forall x, y \in X, \forall h \in H, \frac{fm(hx)}{fm(x)} = \frac{fm(hy)}{fm(y)}$

Proposition 1. For each fuzziness measure fm on X the following statements hold:

- (1). $fm(hx) = \mu(h)fm(x)$, for every $x \in X$;
- (2). $fm(c^-) + fm(c^+) = 1$;
- (3). $\sum_{-q \leq i \leq p, i \neq 0} fm(h_i c) = fm(c)$, $c \in \{c^-, c^+\}$;
- (4). $\sum_{-q \leq i \leq p, i \neq 0} fm(h_i x) = fm(x)$;
- (5). $\sum_{-q \leq i \leq -1} \mu(h_i) = \alpha$ and $\sum_{1 \leq i \leq p} \mu(h_i) = \beta$, where $\alpha, \beta > 0$ and $\alpha + \beta = 1$.

Definition 2. The fuzziness interval of the linguistic terms $x \in X$, denoted by $\mathfrak{I}(x)$, is a subinterval of $[0, 1]$, if $|\mathfrak{I}(x)| = fm(x)$ where $|\mathfrak{I}(x)|$ is the length of $fm(x)$, and recursively determined by the length of x as follows:

- (1). If length of x is equal to 1 ($l(x)=1$), that mean $x \in \{c^-, c^+\}$, then $|\square(c^-)| = fm(c^-)$, $|\square(c^+)| = fm(c^+)$ and $\square(c^-) \leq \square(c^+)$;
- (2). Suppose that n is the length of x ($l(x)=n$) and fuzziness interval $\square(x)$ has been defined with $|\square(x)| = fm(x)$. The set $\{\mathfrak{I}(h_j x) \mid j \in [-q^+ p]\}$, in which $[-q^+ p] = \{j \mid -q \leq j \leq -1 \text{ or } 1 \leq j \leq p\}$, is a partition of $\mathfrak{I}(x)$ and we have: for $h_p x < x$, $\mathfrak{I}(h_p x) \leq \mathfrak{I}(h_{p-1} x) \leq \dots \leq \mathfrak{I}(h_1 x) \leq \mathfrak{I}(h_{-1} x) \leq \dots \leq \mathfrak{I}(h_{-q} x)$; for $h_p x > x$, $\mathfrak{I}(h_{-q} x) \leq \mathfrak{I}(h_{-q+1} x) \leq \dots \leq \mathfrak{I}(h_{-1} x) \leq \mathfrak{I}(h_1 x) \leq \dots \leq \mathfrak{I}(h_p x)$.

3. PROPOSED MODEL

Suppose that $TS(t)$ is a time series that we need forecasting. The model used to forecast $TS(t)$ are built through three procedures as follows:

Fuzzification

Step 0: Convert $TS(t)$ into a variation called $VTS(t)$. Add a integer q into $VTS(t)$ such that all values in this time series are more than 0.

Step 1: Determine the number of linguistic terms, denoted k , used to qualitatively describe values of $TS(t)$.

Step 2:

Determine the universe of discourse of $VTS(t)$, $U = [Dmin - D1, Dmax + D2]$ where $Dmin, Dmax, D1$ and $D2$ are, respectively, minimum, maximum historical values of $VTS(t)$ and proper value which are chosen so that all the values of $VTS(t)$ belong to FU .

Step 3:

Denote $FTS(t)$ is the fuzzy time series generated after doing fuzzification of $TS(t)$. At first, $FTS(t) = \emptyset$.

Use $AX = (X, G, C, H, \leq)$, in which H includes only two hedges, h_{-1} and h_{+1} and $G = \{c^-, c^+\}$, to generate terms.

Let p be FiFo list, Lo and Hi respectively be primary generators, and t be a interger number.

Add Lo and Hi to p ;

$t=2$;

While ($t \leq k$)
 Let x be a linguistic term
 x = first element of p ;
 Use h_{-1} and h_{+1} operate to x in order to generate two new linguistic terms, $h_{-1}x$ and $h_{+1}x$
 Add $h_{-1}x$ and $h_{+1}x$ to rear of p ;
 $t=t+1$;

Sort p in ascending order of semantically ordering relation.

(3) Calculate fuzziness intervals of the linguistic terms. Assign each fuzziness interval to a interval based on U . The intervals create a list of consecutive intervals on U .

(4) Remove the intervals which do not contain any historical values. Suppose that the number of the intervals is m ($m \geq 1$).

(5)
 (a) Find the interval which is leftmost position and containing the maximum number of distinct historical values, suppose that this interval is referred to A_i , to partition it into two sub intervals corresponding two linguistic terms, $h_{-1}A_i$ and $h_{+1}A_i$.

(b) Add two new terms to p .
 (c) Remove the terms that its fuzziness interval do not contain any historical values.

(d) Loop (a), (b) and (c) until get m terms (to get k terms) or do (e) if all the terms's intervals have only one historical value or same historical values.

(e) Do:
 Retake the terms at left position, from right to left, were removed from (4) to get k terms (equal to k intervals). If the number of the left terms is not enough, then get the right terms, from left to right.

(h) Replace each value of $TS(t)$ by an appropriate term getting from p and add the term to $FTS(t)$.

Building relationship groups

Relationships groups are setup following the time points:
 - Build relationship for couples of consecutive values of the $FTS(t)$ at time point t_k where $t_k \leq t$. For convenient, denote each value of the $FTS(t)$ by A_i ($i = 1, 2, \dots$), so each relationship like the following:

$A_i \rightarrow A_j$ where A_i and A_j is, respectively, value of $FTS(t_k-1)$ and $FTS(t_k)$.

- Group relationships having same left side, for instance:

If we have

$A_p \rightarrow A_u,$

$A_p \rightarrow A_q,$

$A_p \rightarrow A_v,$

then the relationships are grouped into $A_p \rightarrow A_u A_q A_v$.

Defuzzifications

Suppose that the value of $VTS(t)$ at tt is qualitatively describe term A_i .

If A_i is left side of relationship $A_i \rightarrow A_j \dots A_k$, then, the forecasting value at $tt+1$ of $VTS(t)$, called ff , is calculated by following formula:

$$ff = \frac{1 \times TB(A_j) + \dots + k \times TB(A_k)}{1 + \dots + k}$$

Where $TB(A_j)$, $TB(A_k)$ respectively is average of historical values falling into fuzziness intervals of $A_j \dots A_k$.

If A_i is left side of relationship $A_i \rightarrow \emptyset$, then ff is $TB(A_i)$.

- Take forecasting values of $VTS(tt+1)$ minus q .

- The forecasting value of $TS(t)$ at $tt + 1$ is computed by mean of following formula:

$$TS(tt+1) = TS(tt) + VTS(tt+1).$$

4. EXPERIMENTAL RESULTS

This section presents experimental results when applying the proposed model to forecast three time series, enrollments at University of Alabama (ALA) from 1971 to 1992, TAIEX (TAI, from 01/12/1992 to 29/12/1992) and Unemployment Index (UNE, from 01/01/2013 to 11/01/2013) in Taiwan [13].

$cov\mathfrak{F}(x)$ and Ld respectively are denoted for mapping of $\mathfrak{F}(x)$ from $[0, 1]$ to the universe of discourse and width of the universe of discourse.

Similar to many papers in the references, this paper uses root of mean squared error (RMSE) to evaluate forecasting quality.

$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i^f - x_i)^2}$, where x_i^f is a forecasting value, x_i is a historical value and n is the number of forecasted values.

This paper uses HA, $AX = (X, G, C, H, \leq)$ in which $G = \{Low, High\}$, $C = \{0, 1, W\}$, $H = \{Very, Little\}$ for applying the proposed model.

Low, High, Very and *Little*, respectively, are denoted by *Lo, Hi, V* and *L* for short. And HisD and PM mean Historical data and Proposed method.

Forecasting result of ALA

The historical values and its variations of ALA time series are shown as below.

Table 1. The values and variations of Alabama

Year	HisD	Values of variations
1971	13055	9
1972	13563	508
1973	13867	304
1974	14696	829
1975	15460	764
1976	15311	-149
1977	15603	292

1978	15861	258
1979	16807	946
1980	16919	112
1981	16388	-531
1982	15433	-955
1983	15497	64
1984	15145	-352
1985	15163	18
1986	15984	821
1987	16859	875
1988	18150	1291
1989	18970	820
1990	19328	358
1991	19337	9

Because 9 is the minimum value of the variations, so place this value to the first position of the variations. Next, take all values of the variations plus 1000 and called the new variations as VT.

Table 2. The variations and changed variations

Values of variations	Values of VT	Fuzzified VT
9	1009	A_4
508	1508	A_6
304	1304	A_6
...
-531	469	A_2
-955	45	A_1
64	1064	A_5
...
820	1820	A_7
358	1358	A_6
9	1009	A_4

We assign VT's universe of discourse to $U = [0, 2400]$, so we have $Ld = 2400$. If we suppose that a value of VT is less

than 14000 that is called *low*, then we can setup following parameters:

$$fm(low) = (2400-1380)/2400 = 0.425, fm(high) = 1-0.425 = 0.575 \text{ and respectively, } cov\mathfrak{F}(low), cov\mathfrak{F}(high): fm(low) \times Ld = 0.425 \times 2400 = 1020, fm(high) \times Ld = 0.575 \times 2400 = 1380.$$

We can assign $\mu(Little) = 0.4, \mu(Very) = 0.6$. From $\mu(Little)$ and $\mu(Very)$ we have $\alpha = 0.4, \beta = 0.6$.

Use the HA to generate 07 linguistic terms: *VVLo, LVLo, VLLo, LLLo, LLH, VLH, VH*. The fuzziness intervals of the terms, respectively, are $[0, 367), [367, 612), [612, 857), [857, 1020), [1020, 1241), [1241, 1572), [1572, 2400]$. These ones are taken as the intervals on the universe of discourse, denoted by $I_k (k = 1, \dots, 7)$.

From Table 2 we have group of relationships in the following:

Table 3. Group of relationships following times

Years	Variations	Time points	Group of relationships
1972	1508	$t = 2$	$A_4 \rightarrow A_6$
1973	1304	$t = 3$	$A_6 \rightarrow A_6$
1974	1829	$t = 4$	$A_6 \rightarrow A_6A_7$
1975	1764	$t = 5$	$A_7 \rightarrow A_7$
1976	851	$t = 6$	$A_7 \rightarrow A_7A_3$
1977	1292	$t = 7$	$A_3 \rightarrow A_6$
1978	1258	$t = 8$	$A_6 \rightarrow A_6A_7A_6$
1979	1946	$t = 9$	$A_6 \rightarrow A_6A_7A_6A_7$
1980	1112	$t = 10$	$A_7 \rightarrow A_7A_3A_5$
1981	469	$t = 11$	$A_5 \rightarrow A_2$
1982	45	$t = 12$	$A_2 \rightarrow A_1$
1983	1064	$t = 13$	$A_1 \rightarrow A_5$
1984	648	$t = 14$	$A_5 \rightarrow A_2A_3$
1985	1018	$t = 15$	$A_3 \rightarrow A_6A_4$
1986	1821	$t = 16$	$A_4 \rightarrow A_6A_7$
1987	1875	$t = 17$	$A_7 \rightarrow A_7A_3A_5A_7$
1988	2291	$t = 18$	$A_7 \rightarrow A_7A_3A_5A_7A_7$
1989	1820	$t = 19$	$A_7 \rightarrow A_7A_3A_5A_7A_7A_7$
1990	1358	$t = 20$	$A_7 \rightarrow$

			A7A3A5A7A7A7A6
1991	1009	$t = 21$	A6 → A6A7A6A7A4
1992	539	$t = 22$	A4 → A6A7A2

The average of the values belonging to I_k ($k = 1, \dots, 7$), respectively, are 45, 504, 749.5, 1012, 1088, 1344 and 1907.

From these values and the group of relationships in Table 3 we determine the forecasting result of VT, after that, take these values minus 1000, next, forecasting result of the variations are obtained.

For example, forecasting value at $t = 22$ is calculated as follows:

Using relationship at $t = 22$, $A4 \rightarrow A6A7A2$, we have forecasting value of VT at this time point: $= (TB(I_6)+2*TB(I_7)+3*TB(I_2))/(1+2+3) = (1344 + 2* 1907 + 3*504)/6 = 1111.52$; $1111.52 - 1000 = 111.52$. The enrollment of 1992 is = enrollment of 1991 + 111.52 = $19337 + 111.52 = 19448.52$.

The forecasting result of ALA, as well as some forecasting results of other models, is presented in following table.

Table 4. Forecasting values of ALABAMA

Year	Lu 2015	Bisht 2016	HN 2019	Tinh 2019	PM
1972	14279	13595.67	13307	13169.5	13399.00
1973	14279	13814.75	14066	13661.09	13907.00
...
1992	19257	19168.56	19589	18421.6	19448.52
1993	N/A	N/A	N/A	18932.2	N/A
RMSE	445.2	428.63	384.34	374.2	278.96

Forecasting result of TAI and UNE

Do the same as [13], we apply the proposed model for forecasting two time series TAI with 07 intervals and UNE with 09 intervals. Forecasting results are printed in the following.

Forecasting result of TAI

Table 5. Forecasting values of TAI

HisD	Wang 2014	Lu 2015	HNV 2016	HN 2019	PM
3635.7	3564.5	3693.1	3709.8	3611.9	3611.90
3614.1	3564.5	3693.1	3709.8	3600.8	3600.80
3651.4	3564.5	3693.1	3709.8	3639	3649.93
...
3742.6	3859.9	3693.1	3709.8	3699.4	3699.40
3696.8	3859.9	3693.1	3709.8	3717.2	3717.20
...
3327.7	3413.3	3519.4	3442.3	3421.1	3421.10
3377.1	3413.3	3519.4	3491.4	3352.6	3352.64
RMSE	107.2	75.7	68.9	39.38	38.43

Forecasting result on UNE

Table 6. Forecasting values of UNE

HisD	Wang 2014	Lu 2015	HNV 2016	HN 2019	PM
7.7	7.62	7.58	7.51	7.70	7.70
7.5	7.62	7.58	7.51	7.55	7.55
...
7.2	7.13	7.07	6.99	7.12	7.12
7.0	7.13	7.07	6.99	7.12	7.12
RMSE	0.19	0.17	0.16	0.10	0.09

5. CONCLUSION

This work introduced a novel fuzzy time series model based on hedge algebras for forecasting time series with three new improvements compared to previous ones. In this model, a new algorithm for determining intervals from the universe of discourse is applied, as well as the relationship groups and weighs following time points are used to determine relationships among fuzzy time series values and forecasting values.

The proposed model was applied for forecasting three time series, enrollments at University of Alabama, TAIEX index and Unemployment rates. Forecasting results were

also compared to some existing methods and it demonstrates that the proposed model gives higher forecasting quality.

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