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Rational Cubic of Timmer and Bezier Curve with Application in Arabic Calligraphy Designs

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ABSTRACT

Curve plays a significant role in Computer Aided Geometric Design (CAGD) because it is easy to model any figure. Bezier curve and Timmer curve are methods that can be used to design any objects or images. In this research, two methods are proposed which are Rational Cubic Bezier and Rational Cubic Timmer to design the Arabic calligraphy. Rational Cubic Bezier and Rational Cubic Timmer are easier to be used in designing since they have an additional parameter which is weight that is used to manipulate images compared to another method which is Cubic Bezier. Timmer curve is also an advanced method to Bezier curve. However, the properties of the curve do not produce the best design compare to Bezier Curve. The comparison between the two methods shows that Rational Cubic Bezier is the best method to be used and able to produce the best design for Arabic Calligraphy.

Key words: Bezier, Timmer, Rational, Arabic Calligraphy

1. INTRODUCTION

Circular arcs are extensively used in the fields of CAGD system and CAD /CAM system which can be represented by rational polynomials. Rational Cubic Timmer (RCT) has been proposed to represent a circular arc. Timmer function was introduced by Harry Timmer of Mc Donnell in the early 1980s. However, the function did not get enough exposure since it did not satisfy the convex hull property. The inventor of Rational Cubic Bezier (RCB) is Dr. Pierre Bezier who was an engineer with Renault car company who set out in the early 1960s to develop a curve formulation for use in shape design [1]. There are several types of rational Bezier curve such as quadratic, cubic, quartic and others [2]. Timmer function actually started with Timmer cubic function before deriving the higher order Timmer blending functions such as quartic, quantic and quintic Timmer functions. Timmer basis function can be used to derive the quartic Timmer function. Next, quartic Timmer function can derive the quintic Timmer function by increasing the power of each bases and number of bases to match the control points that are involved [1].

Generally, it is known that the maximum turning angle for Rational Cubic Bezier is less than and only negative weight conditions can extend its range to but not equal to. Unfortunately, such conditions are not cooperative in CAD system because they lose the convex hull property. That is how Timmer function was produced as an advanced to Bezier function. Although Timmer function does not obey convex hull property, it can be used to make the designing of object easier by manipulating the curve using control point. Moreover, the curve produced for Timmer function is nearer to the control polygon than Bezier function. [3].

Arabic calligraphy is mostly recognized as Islamic calligraphy where the work is fundamentally based on the Quran by the craftsmanship of renowned calligraphers. Arabic calligraphy has evolved and developed into multiple scripts and each of it can be recognized by its peculiar style and usage. It is a highly respected art form with hundreds of different styles including a few essential scripts such as the Kufi, Naskh, Nasta'alik, Riq'a, Thuluth and the Diwani [4]. Thuluth has cursive script that has large, elongated and elegant letters. It is one of the most difficult writing to master. This script was invented in the 11th century but went through an evolution in the 15th century during the Ottoman Empire. Thuluth script is mostly used on mosques decorations [5]. In this paper, Thuluth was used as reference figure in generating the Arabic calligraphy.

The traditional method to create a design is quite hard and needs a very good skill in designing the calligraphic font. It is also complicated to make the tools such as calligraphy pen that called as 'kalem' where the ink is made from a lot of ingredients such as soot of hemp oil, gum Arabic and others. Since calligraphy is the art of forming beautiful symbol, these methods were introduced as an alternative tool that can help the calligraphy designers to show their skills with their own creativity easily by using the technology of computer. The problem for those who are interested in this field but do not have the skills to use the traditional method can now be solved easily by only using the skills of computer. Furthermore, it also can save their time and budget from buying those expensive tools to create the design as they wanted. This

paper was purposed to discuss the comparison of Rational Bezier and Rational Timmer curve, by applying the methods in designing Arabic calligraphic and comparing the best designs produced from the manipulation that have been done. The main objective of this paper to applied both methods in designing the Arabic Calligraphy chosen and to compare the best design produced form these methods.

2. METHODOLOGY

2.1 Rational Cubic Timmer Curves

According to [1], Timmer function was introduced by Harry Timmer. However, it did not get enough exposure since it did not satisfy one of the properties which is convex hull property. This function is an advanced of Bezier function. The number of control point in Cubic Timmer is same as Cubic Bezier. Although Timmer function does not obey the convex hull property, we can use it in designing easier. The curve produced is nearer to the control polygon and apparently for cubic Timmer function, it cuts the control polygon. It is easy to manipulate the curve by manipulating the value of weight as we were going to demonstrate some designs.

The basis function for Cubic Timmer are stated as in (1), (2), (3) and (4). Figure 1 shows the graph of basis function for Cubic Timmer and Figure 1 shows the Rational Cubic Timmer curve.

$$B_0(t) = (1-t)^2(1-2t) \quad (1)$$

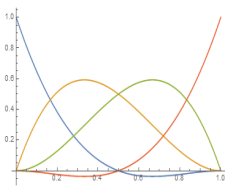
$$B_1(t) = 4t(1-t)^2 \quad (2)$$

$$B_2(t) = 4t^2(1-t) \quad (3)$$

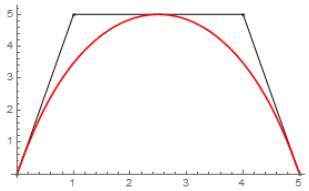
$$B_3(t) = t^2[1-2(1-t)] \quad (4)$$

The expression used for Rational Cubic Timmer is the same as Rational Cubic Bezier but with different basis. The equation for rational Bezier curve is noted as [3].

$$r(t) = \frac{\sum_{i=0}^{n} w_i P_i B_i(t)}{\sum_{i=0}^{n} w_i B_i(t)} \quad ,0 \le t \le 1 \quad (5)$$



(a) Graph of basis function



(b) Graph of Curve

Figure 1: Graph of basis function and curve of cubic Rational Timmer

3.2 Rational Cubic Bezier Curve

Rational Bezier curve has the blending functions that are rational polynomials. It was noted that if all the value of weight is same for all the control point, a rational Bezier curve reduces to a polynomial Bezier curve. In addition, rational Bezier curves have several advantages as compared to polynomial Bezier curves. Rational Bezier curves provide more control over the shape of a curve than a polynomial Bezier curve [6].

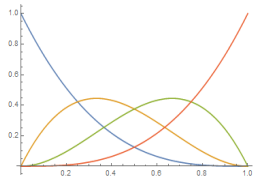
According to [3], if $w_0 = w_3 = 1$ and positive number for w_1 and w_2 for rational cubic Bezier curve, the basis function for Rational Cubic Bezier is the same as Cubic Bezier and are shown in equation (6) to (9) and the equation for rational Bezier curve is noted as equation (5), [2]. The basis and curve of this function are shown in Figure 2.

$$B_0(t) = (1-t)^2[1-t] \quad (6)$$

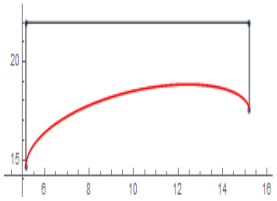
$$B_1(t) = 3t(1-t)^2 \quad (7)$$

$$B_2(t) = 3t^2(1-t) \quad (8)$$

$$B_3(t) = t^2[1-(1-t)] \quad ,t \in [0,1] \quad (9)$$



(a) Graph of basis function



(b) Graph of Curve

Figure 2: Graph of basis function and curve of cubic Rational Bezier

3. APPLICATION IN ARABIC CALLIGRAPHY DESIGN

In this paper, Arabic Calligraphy was chosen to be design by using Rational Cubic Bezier and Rational Cubic Timmer. The result will be compared to determine which method can produce the best design based on the smoothest curve and closest to the real figure. All the data from this research are secondary data since it was directly collected from the selected image. Figure 3 shows the real figure that was used as a reference to design the Arabic Calligraphy [7]. The design is known as Thuluth and it is not too complicated and not so simple to be designed by using those two methods in this paper.

Finding control points was the first step that has be done before designing the Arabic Calligraphy for each method. The figure of Arabic Calligraphy that was chosen will be printed on the graph paper and divided it into several segments to find the control point. Four control points ere used for each segment since the curve was degree three. Control polygon

then will be sketched based on the control point. Then, all the control polygon for each segment were combined to form a complete Arabic calligraphy design. Figure 4 shows the sketch of control polygon from the real figure on the graph paper and 27 curves were defined from the real figure and all the curves were generated using Mathematica software.

Figure 3: The Real Figure
***Source**: Arabic calligraphy: A computational exploration (2001)*

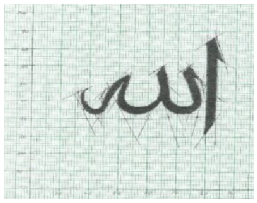


Figure 4: The sketch of control polygon from the real figure on the graph paper

Table 1: The value of weighted

Curve	Weights
Curve 1	w0 = 2 ; w1 = 2 ; w2 = 2 ; w3 = 2
Curve 2	w4 = 1; w5 = 1 ; w6 = 1 ; w7 = 1
Curve 3	w8 = 3 ; w9 = 3 ; w10 = 3 ; w11 = 3
Curve 4	w12 = 5 ; w13 = 5 ; w14 = 5 ; w15 = 5
Curve 5	w16 = 6 ; w17 = 6 ; w18 = 6 ; w19 = 6
Curve 6	w20 = 7 ; w21 = 7 ; w22 = 7 ; w23 = 7
Curve 7	w24 = 4 ; w25 = 4 ; w26 = 4 ; w27 = 4
Curve 8	w28 = 2 ; w29 = 2 ; w30 = 2 ; w31 = 2
Curve 9	w32 = 5 ; w33 = 5 ; w34 = 5 ; w35 = 5
Curve 10	w36 = 6 ; w37 = 6 ; w38 = 6 ; w39 = 6
Curve 11	w40 =8 ; w41 = 8 ; w42 = 8 ; w43 =8
Curve 12	w44 = 7 ; w45 = 7 ; w46 = 7 ; w47 = 7
Curve 13	w48 = 9 ; w49 = 9 ; w50 = 9 ; w51 = 9
Curve 14	w52 =9 ; w53 = 9 ; w54 = 9 ; w55 = 9
Curve 15	w56 = 1 ; w57 = 1 ; w58 = 1 ; w59 = 1
Curve 16	w60 = 2 ; w61 =2 ; w62 =2 ; w63 = 2
Curve 17	w64 = 2 ; w65 = 2 ; w66 = 2 ; w67 = 2
Curve 18	w68 = 1 ; w69 = 1 ; w70 = 1 ; w71 = 1
Curve 19	w72 = 2 ; w73 = 2 ; w74 = 2 ; w75 = 2
Curve 20	w76 = 4 ; w77 = 4 ; w78 = 4 ; w79 = 4
Curve 21	w80 = 8 ; w81 = 8 ; w82 = 8 ; w83 = 8
Curve 22	w84 = 7 ; w85 = 7 ; w86 = 7 ; w87 = 7
Curve 23	w88 = 3 ; w89 = 3 ; w90 = 3 ; w91 = 3
Curve 24	w92 = 3 ; w93 = 3 ; w94 = 3 ; w95 = 3
Curve 25	w96 = 3 ; w97 = 3 ; w98= 3 ; w99 = 3
Curve 26	w100 = 4 ; w101 = 4 ; w102 = 4 ; w103 = 4
Curve 27	w104 = 5 ; w105 = 5 ; w106 = 5 ; w107 = 5

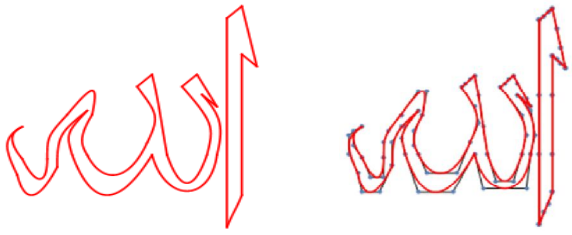


Figure 5: The Arabic calligraphy of Rational Timmer curve

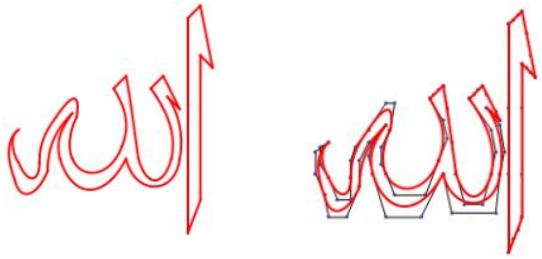
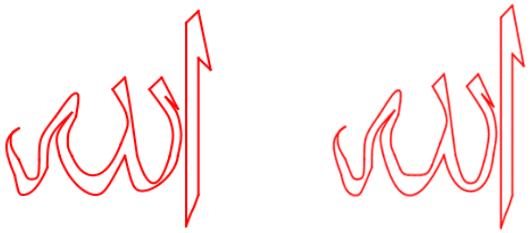


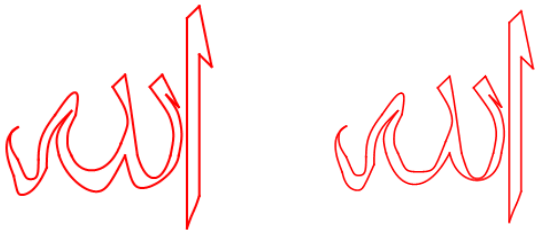
Figure 6: The Arabic calligraphy of Rational Bezier curve

Figure 5 and figure 6 show the Arabic Calligraphy obtained from Rational Cubic Timmer and Bezier Curve by using control point defined and the values weight that are shown in Table 1. The figure shows the designs are different because of the properties of the curve. The Rational Timmer curve produced the design detach to the middle of control polygon while Rational Bezier curve was far from the control polygon. Based on this property, the manipulation of the values of weight was done to observe the effect of the curves to the designs produced.

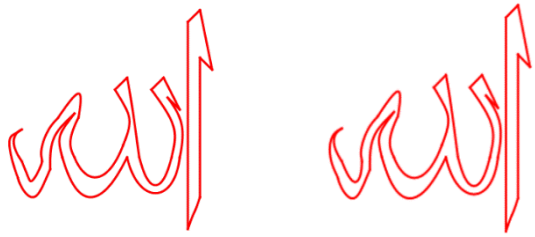
3.1 Manipulation the Values of Weighted



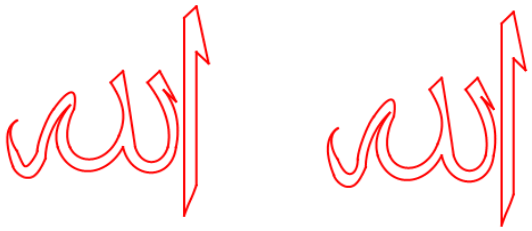
Design (a) **Design (b)**
Figure 7: The Arabic calligraphy of Rational with manipulation of middle values of weight



Design (c) **Design (d)**
Figure 8: The Arabic calligraphy of Rational Bezier with manipulation of middle values of weight



Design (e) **Design (f)**
Figure 9: The Arabic calligraphy of Rational Timmer with manipulation of all values of weight



Design (g) **Design (h)**

Figure 10: The Arabic calligraphy of Rational Bezier with manipulation of all values of weight

Figure 7 until Figure 10 are the result based on manipulations of the middle values of weight and all values of weight. There are 8 designs generated from the manipulation of the weighted. The results show the designs changed when the value of weight was changed, and it produced varieties of designs. Both methods can be used as an alternative method in designing the Arabic Calligraphy and by manipulating the values of weight, it will help the designer to produce varieties of designs easily, with effective time and cost. Both designs have their own uniqueness of designs and it's also can be applied in other 2 dimensional designs.

Based on both manipulations, Figure 9 and 10 show the best result as compared to Figure 7 and 8 because all the values of weight were manipulated in order to get the best designs. Meanwhile, the comparison also can be made based on two methods and it is shown that the Rational Bezier curve produced the closest design to the actual figure and the designs is smooth as compared to Rational Timmer curve. Here it shows that, the Rational Bezier curve was produced better designs as compared to Rational Timmer curve.

4. CONCLUSION

This paper focuses on Rational Cubic Timmer that has similar parameters as in Rational Cubic Bezier which are control point and weight to manipulate the shape of curve. The manipulation of control point and weight is also the same as in Rational Cubic Bezier. However, the design generated produced a bit difference from Rational Cubic Bezier because it produced a curve that is detached to the control polygon. The different type of Arabic Calligraphy designs was generated for different methods used in this research. This is because each method has their own properties and parameter used which affect the shape of curve for the design.

As a conclusion, it is found that Rational Cubic Bezier is the best method that can be used to design Arabic Calligraphy because it can generate the best design since the curve produced is smoother as compared to Rational Cubic Timmer and it also able to generate the design which is close to the real figure. Rational Cubic Bezier is also better than Cubic Bezier polynomial because the method is easier to use since it has additional parameter to manipulate the curve rather than using only control point as in Cubic Bezier.

This paper was proposed as an alternative way to design an Arabic Calligraphy by using two methods which are Rational Cubic Bezier and Rational Cubic Timmer. The methods used in this research are from Computer Aided Geometric Design (CAGD) which also contains more methods, provided that they can be used for future research. The methods not only can be used for Arabic design but also for other designs and objects which can be done in 3D which is different from this research that was designed in 2D. In other papers, it is stated that the higher the degree, the better design would be generated. Thus, a higher degree for those two methods can be used in future research for any type of design.

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