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An Interactive Method for Solving Fuzzy Multi-objective Linear Programming Problems

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ABSTRACT
This paper is concerned with multi-objective linear programming problems in which the coefficients are expressed as fuzzy numbers of triangular type. An interactive method, to enhance the weights in the weighted sum problem, is introduced. In the scalarized problem, the weights are determined via the ideal minimum and maximum values of the objective functions. A numerical example is given to clarify the presented method.

Key words: Fuzzy set, interactive method, multi-objective linear programming.

1. INTRODUCTION

In real-world, the optimization problems often involve two or more objectives to be optimized at the same time. These objectives are called objective functions and they are usually conflicting. This means that we cannot improve one of them without worsening the others. We call this type of problems multi-objective programming (MOP) problems or vector optimization problems (VOPs). As a matter of fact, in contrast with single objective optimization problems, MOP problems have many Pareto-optimal solutions or efficient solutions with different trade-offs. The set of these solutions is called Pareto-optimal set and its corresponding set in the objective space is called the Pareto frontier [1]–[6]. In MOP problems, there is a need for a decision maker (DM) to compare between different efficient solutions and choose the preferred one amongst them.

It is known that the DM can engage in solving MOP problems in three different ways [7], [8]. In the first way (a priori method), the DM provides his/her preferences before the process of solution. While in the second way (a posteriori method), the DM selects the final solution after providing him/her with a set of efficient solutions. In the third way, the DM provides preferences during the solution process and this is called an interactive method [9], [10].

Fuzzy multi-objective programming (F-MOP) problems are MOP problems in which the coefficients in their objective functions and/or constraints are fuzzy numbers [11]–[13]. There are many applications of this type of problems such as power markets, business management and energy system [14]–[16]. For solving fuzzy multi-objective linear programming (F-MOLP) problems, the authors of [11], [17], [18] and other researchers used goal programming approach. Khalifa and Al-Shabi [19] introduced an interactive method to solve fuzzy multi-objective assignment problem which is a type of multi-objective linear programming problems.

In this paper, we introduce an interactive method to improve the weights in the weighting sum technique for solving (F-MOLP) problems with triangular fuzzy numbers in the objective functions and constraints. The rest of this paper is organized as follows; section 2 shows some important and basic definitions. In section 3, the F-MOLP problem is formulated. Section 4 presents the proposed interactive algorithm for solving the F-MOLP problems. A numerical example is introduced in section 5. Summary and conclusion are provided in section 6.

2. PRELIMINARIES

Some paramount definitions and basic notions of the fuzzy set theory are introduced, see [9].

Definition 1. A fuzzy set $\tilde{\Lambda}$ can be defined by the ordered pairs $\{(x, \mu_{\tilde{\Lambda}}(x)) \mid x \in R\}$, where R is the real numbers set and $\mu_{\tilde{\Lambda}}(x) \in [0,1]$ is called the fuzzy set membership function.

Definition 2. A convex fuzzy set ($\tilde{\Lambda}$) is a fuzzy set in which: $\mu_{\tilde{\Lambda}}(\lambda x + (1-\lambda)z) \geq \min\left[\mu_{\tilde{\Lambda}}(x), \mu_{\tilde{\Lambda}}(z)\right]$, for all $x, z \in R$ and for all $\lambda \in [0,1]$.

Definition 3. A fuzzy number ($\tilde{\Lambda}$) is a convex fuzzy subset of the real line R with membership function $\mu_{\tilde{\Lambda}}(x): R \to [0,1]$. This fuzzy number is called positive if $\mu_{\tilde{\Lambda}}(x)$=0, for all $x \leq 0$.

Definition 4. A triangular fuzzy number ($\tilde{\Lambda}$) that is represented by three real parameters (a,b,c) such that

$a \le b \le c$, see (Figure.1), is defined as $\tilde{\Lambda} = (x, \mu_{\tilde{\Lambda}}(x))$ where:

$$\mu_{\tilde{\Lambda}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & otherwise \end{cases} \tag{1}$$

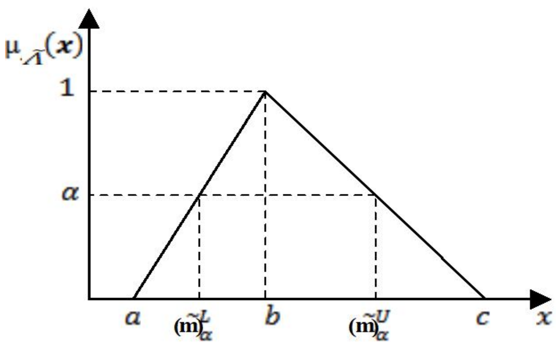

Figure 1: Triangular Fuzzy Number

Definition 5. The α-level (or α-cut) set of a fuzzy number $\tilde{\Lambda}$, denoted by $(\tilde{\Lambda})_{\alpha}$, is defined as the crisp set for which the degree of its membership functions is equal to or greater than a number α∈[0,1] such that $(\tilde{\Lambda})_{\alpha} = \{x | \mu_{\tilde{\Lambda}}(x) \ge \alpha\}$.

The α-cut set of a triangular fuzzy number $\tilde{m}$ is $(\tilde{m})_{\alpha} = [(\tilde{m})_{\alpha}^{L}, (\tilde{m})_{\alpha}^{U}]$, where $(\tilde{m})_{\alpha}^{L} = (1-\alpha)a + \alpha b$ and $(\tilde{m})_{\alpha}^{U} = (1-\alpha)c + \alpha b$ represent the lower and upper bounds (cuts), respectively (Figure 1).

3. PROBLEM FORMULATION

The fuzzy multi-objective linear programming (F-MOLP) problem of minimization type can be formulated as follows:

(F-MOLP) $\min \tilde{F}(x) = \{\tilde{f}_1(x), \tilde{f}_2(x), \ldots, \tilde{f}_n(x)\}$ (2)

subject to

$$x \in X = \{x \in R^N \mid \tilde{A}x \begin{pmatrix} \le \\ \ge \end{pmatrix} \tilde{b}, x \ge 0, b \in R^k\} \tag{3}$$

where: $\tilde{f}_i(x) = \tilde{C}_i^T x : R^N \to R$, $i = 1, 2, \ldots, n$ is the i^{th} objective function, $\tilde{C}_i = (\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_N)$ is a coefficient vector, $x \in R^N$ is the vector of the decision variables $x = (x_1, x_2, \ldots, x_N)$. k is the number of constraints, $\tilde{A}$ is the coefficient matrix of size $k \times N$, and X is the convex decision space (feasible region).

For a certain value of α, the objective function of minimization type $\tilde{f}_i(x)$, $i = 1, 2, \ldots, n$ can be replaced by the lower bound of its α-cut [17], [20].

This means, $(\tilde{f}_i(x))_{\alpha}^{L} = (\tilde{C}_i^T)_{\alpha}^{L} x$, $(i = 1, 2, \ldots, n)$ (4)

Also, the maximization-type objective function $\tilde{f}_i(x)$, $i = 1, 2, \ldots, n$ can be replaced by the upper bound of its α-cut.

This means, $(\tilde{f}_i(x))_{\alpha}^{U} = (\tilde{C}_i^T)_{\alpha}^{U} x$, $(i = 1, 2, \ldots, n)$ (5)

The constraints,

$$\left.\begin{aligned} \tilde{A}_j x \ge \tilde{b}_j, \quad (j = 1, 2, \ldots, t), \\ \tilde{A}_j x \le \tilde{b}_j, \quad (j = t+1, \ldots, k) \end{aligned}\right\} \tag{6}$$

can be rewritten as follows [11]:

$$\left.\begin{aligned} (\tilde{A}_j)_{\alpha}^{U} x \ge (\tilde{b}_j)_{\alpha}^{L}, \quad (j = 1, 2, \ldots, t), \\ (\tilde{A}_j)_{\alpha}^{L} x \le (\tilde{b}_j)_{\alpha}^{U}, \quad (j = t+1, \ldots, k) \end{aligned}\right\} \tag{7}$$

Now, for a certain value of α, the minimization type F-MOLP problem will be denoted as (α-MOLP) problem:

(α-MOLP)

$$\min (\tilde{F}(x))_{\alpha}^{L} = \{(\tilde{f}_1(x))_{\alpha}^{L}, (\tilde{f}_2(x))_{\alpha}^{L}, \ldots, (\tilde{f}_n(x))_{\alpha}^{L}\} \tag{8}$$

subject to

$$\left.\begin{aligned} &(\tilde{A}_j)_{\alpha}^{U} x \ge (\tilde{b}_j)_{\alpha}^{L}, (j = 1, 2, \ldots, t), \\ &(\tilde{A}_j)_{\alpha}^{L} x \le (\tilde{b}_j)_{\alpha}^{U}, (j = t+1, \ldots, k), \\ &x \ge 0. \end{aligned}\right\} \tag{9}$$

Definition 6. [10] A point $\overline{x}$ is called an α-Pareto optimal solution to the (α-MOLP) problem, if and only if there is no another x such that $(\tilde{f}_i(x))_{\alpha}^{L} \le (\tilde{f}_i(\overline{x}))_{\alpha}^{L}$ with strict inequality holding for at least one i .

The (α-MOLP) problem is a deterministic (non-fuzzy) MOLP problem and it can be solved by applying the weighting sum approach to convert it to a corresponding single objective programming problem as follows:

$$\min Z = \sum_{i=1}^{n} \omega_i (\tilde{f}_i(x))_{\alpha}^{L} \tag{10}$$

subject to the constraints (9).

where ω_i is the weight of the corresponding objective function $(\tilde{f}_i(x))_{\alpha}^{L}$. Following [7] and [11], let v_i, $i = 1, 2, \ldots, n$ be the individual minimum solution of $(\tilde{f}_i(x))_{\alpha}^{L}$ where:

$$v_i = \min_{x \in X} [(\tilde{f}_i(x))_{\alpha}^{L} = (\tilde{C}_i^T)_{\alpha}^{L} x], \quad (i = 1, 2, \ldots, n) \tag{11}$$

Similarly, Y_i , $i=1,\ 2,\ \dots,\ n$ denotes the individual maximum solution of $\left(\tilde{f}_i(x)\right)_\alpha^U$ where:

$$\mathrm{Y}_i=\max_{x\in X}[\left(\tilde{f}_i(x)\right)_\alpha^U=\left(\tilde{C}_i^T\right)_\alpha^U x],\ \ (i=1,\ 2,\ \dots,\ n) \tag{12}$$

The weights ω_i in (10) can be calculated as follows:

$$\omega_i=\frac{\mathrm{Y}_i-v_i}{\sum_{i=1}^{n}\left(\mathrm{Y}_i-v_i\right)},\ \sum_{i=1}^{n}\omega_i=1,\ (i=1,\ 2,\ \dots,\ n) \tag{13}$$

The following section presents an interactive algorithm which can be followed to improve the weights in the scalarization problem.

4. INTERACTIVE ALGORITHM

The proposed interactive algorithm and solution procedure for solving the F-MOLP problem can be summarized in the following steps:

Step 1. In cooperation with the DM, set the value of α∈[0,1] in order to formulate the (α -MOLP) problem in (8) and (9).

Step 2. Obtain (v_i): the individual minimum solution (the lower bound) of the objective functions by solving the problem (11).

Step 3. Obtain (Y_i): the individual maximum solution (the upper bound) of the objective functions by solving the problem (12).

Step 4. Evaluate the respective numerical weights ω_i using relation (13).

Step 5. Formulate the corresponding weighting sum problem (10) and solve it using simplex method to get the solution ($\overline{x}^1$).

Step 6. The DM compares $\left(\tilde{f}_i\left(\overline{x}^1\right)\right)_\alpha^L$ with the ideal solutions (v_i). If the DM is satisfied with them, go to step 8, otherwise go to step 7.

Step 7. Find the value of the upper bound of each objective function at ($\overline{x}^1$) i.e., $\left(\tilde{f}_i\left(\overline{x}^1\right)\right)_\alpha^U$ and take $\left(\tilde{f}_i\left(\overline{x}^1\right)\right)_\alpha^L$ as a new upper bound (Y_i), then return to step 4.

Step 8. Stop.

5. ILLUSTRATIVE EXAMPLE

Consider the following (F-MOLP) problem:

$$\min\left\{\tilde{f}_1(x),\tilde{f}_2(x),\tilde{f}_3(x)\right\}$$

where:

$$\tilde{f}_1(x)=\left(x_1+\tilde{3}x_2+\tilde{2}x_3+\tilde{3}x_4\right),\ \tilde{f}_2(x)=\left(\tilde{2}x_1+\tilde{9}x_2+\tilde{3}x_3+\tilde{5}x_4\right)$$

and $\tilde{f}_3(x)=\left(\tilde{3}x_1+\tilde{9}x_2+\tilde{9}x_3+x_4\right)$.

subject to

$$\tilde{3}x_1-x_2+x_3+\tilde{3}x_4\le\tilde{48},\ \tilde{2}x_1+\tilde{4}x_2+\tilde{2}x_3-\tilde{2}x_4\le\tilde{35},$$

$$x_1+\tilde{2}x_2-x_3+x_4\ge\tilde{30},\ x_1,x_2,x_3,x_4\ge 0.$$

The triangular-type fuzzy numbers are given as follows:

$\tilde{2}=(0,2,3)$, $\tilde{3}=(2,3,4)$, $\tilde{4}=(3,4,5)$, $\tilde{5}=(4,5,6)$,

$\tilde{6}=(5,6,7)$, $\tilde{8}=(6,8,10)$, $\tilde{9}=(8,9,10)$,

$\tilde{30}=(28,30,32)$, $\tilde{35}=(33,35,37)$, $\tilde{48}=(45,48,49)$.

Then, for an α-cut, the corresponding (α-MOLP) problem will be:

$$\min\begin{cases}\left(\tilde{f}_1(x)\right)_\alpha^L = x_1+(2+\alpha)x_2+(2\alpha)x_3+(2+\alpha)x_4\\ \left(\tilde{f}_2(x)\right)_\alpha^L=(2\alpha)x_1+(8+\alpha)x_2+(2+\alpha)x_3+(4+\alpha)x_4\\ \left(\tilde{f}_3(x)\right)_\alpha^L = (2+\alpha)x_1+(8+\alpha)x_2+(8+\alpha)x_3+x_4\end{cases}$$

subject to

$$(2+\alpha)x_1-x_2+x_3+(2+\alpha)x_4\le(49-\alpha),$$

$$(2\alpha)x_1+(3+\alpha)x_2+(2\alpha)x_3-(2\alpha)x_4\le(37-2\alpha),$$

$$x_1+(3-\alpha)x_2-x_3+x_4\ge(28+2\alpha),x_1,x_2,x_3,x_4\ge 0$$

Step1. For α=0.6, the (0.6-MOLP) problem is:

$$\min\begin{cases}\left(\tilde{f}_1(x)\right)_{0.6}^L = x_1+2.6x_2+1.2x_3+2.6x_4\\ \left(\tilde{f}_2(x)\right)_{0.6}^L=1.2x_1+8.6x_2+2.6x_3+4.6x_4\\ \left(\tilde{f}_3(x)\right)_{0.6}^L = 2.6x_1+8.6x_2+8.6x_3+x_4\end{cases}$$

subject to

$$X=\{x=(x_1,x_2,x_3,x_4)|2.6x_1-x_2+x_3+2.6x_4\le 48.4,$$

$$1.2x_1+3.6x_2+1.2x_3-1.2x_4\le 35.8,\ \ x_1+2.4x_2-x_3+x_4\ge 29.2,$$

$$x_1,x_2,x_3,x_4\ge 0\}.$$

Step 2. The individual minimum (v_i) of each objective function is obtained by solving the above lower bound model for each objective function individually. The values are shown in Table 1.

Table 1: Individual minimum optimum values

Objective function	$\left(\tilde{f}_1(x)\right)_{0.6}^L$	$\left(\tilde{f}_2(x)\right)_{0.6}^L$	$\left(\tilde{f}_3(x)\right)_{0.6}^L$
Minimum (v_i)	31.279	59.583	52.767

Step 3. The individual maximum (Y_i) of each objective function is obtained by solving each of the following upper bound problems:

$$\max_{x\in X}\left(\tilde{f}_1(x)\right)_{0.6}^{U} = x_1 + 3.4x_2 + 2.4x_3 + 3.4x_4,$$

$$\max_{x\in X}\left(\tilde{f}_2(x)\right)_{0.6}^{U} = 2.4x_1 + 9.4x_2 + 3.4x_3 + 5.4x_4,,$$

$$\max_{x\in X}\left(\tilde{f}_3(x)\right)_{0.6}^{U} = 3.4x_1 + 9.4x_2 + 9.4x_3 + x_4.$$

The maximum values are shown in Table 2.

Table 2. Individual maximum optimum values

Objective function	$\left(\tilde{f}_1(x)\right)_{0.6}^{U}$	$\left(\tilde{f}_2(x)\right)_{0.6}^{U}$	$\left(\tilde{f}_3(x)\right)_{0.6}^{U}$
Maximum (Y_i)	150.5	313.127	254.8734

Step 4. The weights will be:

$\omega_1 = 0.20739$, $\omega_2 = 0.441$, and $\omega_3 = 0.35161$.

Step 5. Solve:

$$\min Z = 1.650776x_1 + 7.35566x_2 + 4.419314x_3 + 2.919424x_4$$

subject to $x \in X$.

By using the MATLAB (version R2018b) program to solve the above linear programming problem, the optimal solution is $\overline{x}^1 = (19.254, 3.801, 0, 0.824)$.

Step 6. Ask the DM to compare $\left(\tilde{f}_1(\overline{x}^1)\right)_{0.6}^{L} = 31.279$, $\left(\tilde{f}_2(\overline{x}^1)\right)_{0.6}^{L} = 59.584$, and $\left(\tilde{f}_3(\overline{x}^1)\right)_{0.6}^{L} = 83.573$ with the individual minimum optimal solutions (v_i) in Table1. Assume that the DM is not satisfied with this solution as he/she wants to decrease the value of $\left(\tilde{f}_3(x)\right)_{0.6}^{L}$ to be less than 83.753.

Step 7. We get $\left(\tilde{f}_1(\overline{x}^1)\right)_{0.6}^{U} = 34.979$, $\left(\tilde{f}_2(\overline{x}^1)\right)_{0.6}^{U} = 86.389$ and $\left(\tilde{f}_3(\overline{x}^1)\right)_{0.6}^{U} = 102.017$, and set $\left(\tilde{f}_1(x)\right)_{0.6}^{L}$, $\left(\tilde{f}_2(x)\right)_{0.6}^{L}$ and $\left(\tilde{f}_3(x)\right)_{0.6}^{L}$ as the new upper bounds Y_1, Y_2 and Y_3, respectively.

Hence, the new weights can be computed from (13) as follows:

$\omega_1 = 0$, $\omega_2 = 0$ and $\omega_3 = 1$.

The new problem will be:

$$\min Z = 2.6x_1 + 8.6x_2 + 8.6x_3 + x_4$$

subject to

$x \in X$,

$\left(\tilde{f}_1(x)\right)_{0.6}^{L} \le \left(\tilde{f}_1(\overline{x}^1)\right)_{0.6}^{U}$ i,e.,

$x_1 + 2.6x_2 + 1.2x_3 + 2.6x_4 \le 34.979$,

and $\left(\tilde{f}_2(x)\right)_{0.6}^{L} \le \left(\tilde{f}_2(\overline{x}^1)\right)_{0.6}^{U}$ i.e.,

$1.2x_1 + 8.6x_2 + 2.6x_3 + 4.6x_4 \le 86.384$.

Thus, we get the following optimal solution $\overline{x}^2$ =(16.941, 3.801, 0, 3.137). Ask the DM to compare $\left(\tilde{f}_1(\overline{x}^2)\right)_{0.6}^{L}$ =34.97, $\left(\tilde{f}_2(\overline{x}^2)\right)_{0.6}^{L}$ =67.44, and $\left(\tilde{f}_3(\overline{x}^2)\right)_{0.6}^{L}$ =79.87 with the individual minimum optimal solutions (v_i) in Table1. Assume that the DM is satisfied with this solution. Then, stop, and the Pareto-optimal solution of the (α-MOLP) problem at α=0.6 is

$\overline{x} = (16.941, 3.801, 0, 3.137)$.

In step 7 of this example, $\omega_1 = \omega_2 = 0$ and to avoid the problem in which $\overline{x}^2 = \overline{x}^1$, the two constraints $\left(\tilde{f}_1(x)\right)_{0.6}^{L} \le \left(\tilde{f}_1(\overline{x}^1)\right)_{0.6}^{U}$ and $\left(\tilde{f}_2(x)\right)_{0.6}^{L} \le \left(\tilde{f}_2(\overline{x}^1)\right)_{0.6}^{U}$ have been added to the set of constraints X. This procedure has given us a new solution ($\overline{x}^2$).

6. CONCLUSION

This paper is preoccupied with the fuzzy multi-objective linear programming (F-MOLP) problem. Fuzzy numbers have been used to express the lack of information existing in practice. Using the concept of α-level sets of fuzzy numbers, the corresponding deterministic α-multi-objective linear programming problem has been obtained. In order to produce the numerical weights included in the weighting sum approach, an interactive algorithm has been presented. This algorithm allows the DM to participate in the steps of finding the optimal solution. An illustrative example has been given.

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