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Optimal consensus of a multi-agent networked systems

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Abstract: **Optimal consensus for a network of multi-agent systems is one of the most important researched topics due to its importance in Network Analysis of Systems under Dynamic conditions. In this paper the problem is defined explicitly and a topological graph for the network is constructed. Then the Linear Quadratic Regulator Method (LQR) and the Linear Matrix Inequality (LMI) Approach are considered and they are used to achieve consensus. A study about the procedures and their behavior is carried out with simulation of the six vehicle network for the velocity parameter using the robust control toolbox of MATLAB.**

Keywords: **Consensus, Linear Matrix Inequality, Linear Quadratic Regulator Method, State Space.**

I. INTRODUCTION

A fully autonomous system must have the capacity to perform hardware repair, if one of its components fails. The autonomous systems must perform well under significant uncertainties in the plant and environment for extended periods of time under the effect of disturbances by compensating for system failures due to external interventions. Generally such systems are highly advanced and are used in controlled conditions. Further, most natural systems have autonomous nature as an internal characteristic When multi-agent systems are networked with a view of autonomous nature , as one of the major driving factors it can be considered as an autonomous network of those multi-agent systems. Such systems have an advantage in that they are self intelligent and correcting in working towards a singularity. Some of the best examples are neural networks of higher dimensions like human brain, Artificial intelligence based systems, Expert systems. Examples of these systems are often found in satellites, manoeuvres of a group of unmanned aerial vehicles (UAVs) for intelligence, surveillance and reconnaissance (ISR) missions. The paper compares the best optimal methods with widespread use and provides the distinction between them.

II. PROBLEM DEFINATION

A. Model Description

Multi-agent teams: Consider the set of agents E= {i =1, 2, 3 ….N}, Where, N=number of agents.

Further, each member of the team is governed by the dynamical representation system is

$$\dot{X}_i = A_i X_i + B_i\, U_i,\ X_i \in R^n\ ;\ U_i \in R^m\ , i=1,2,...N \qquad (1)$$

$$Y_i = C_i X_i,\ \ Y_i \in R^q;\ i=1,2,3\ldots\ldots N \qquad (2)$$

Where,

X_i= state vector, U_i= input vector, Y_i= output vector

Matrices A_i, B_i, C_i have the required dimensions.

Variables n, m and q mean the dimensions of the state, input and output vectors of the respective agents.

For the entire team the equations can be written as

$\dot{X} = AX + BU$ and $Y = CX$

So,

$$X_{Nn \times 1} = [(X_1)^T \ldots\ldots\ldots\ldots\ldots\ldots\ldots (X_N)^T]$$
$$U_{Nm \times 1} = [(U_1)^T \ldots\ldots\ldots\ldots\ldots\ldots\ldots (U_N)^T]$$
$$Y_{Nq \times 1} = [(Y_1)^T \ldots\ldots\ldots\ldots\ldots\ldots\ldots (Y_N)^T]$$

The matrices can be defined thus as

$$A=\text{Diag}\ \{A_1 \ldots\ldots\ldots\ldots\ldots\ldots A_N\}$$
$$B=\text{Diag}\ \{B_1 \ldots\ldots\ldots\ldots\ldots\ldots B_N\}$$
$$C=\text{Diag}\ \{C_1 \ldots\ldots\ldots\ldots\ldots\ldots C_N\}$$

B. Problem Statement: Consensus in a Team of Multi-Agents [1]

Our main goal is to ensure agent's state converge to the same value, i.e. for all Xi → Xj. It is desired that the team reaches to a consensus in the subspace spanned by the vector **1**,

$X_{ss} = [(X_1)^T_{ss} \ldots (X_N)^T_{ss}]^T = [1\ 1 \ldots 1]^T \varnothing\ \omega_{ss} = \mathbf{1}\varnothing\omega_{ss}$

where, ω_{ss} is the final state vector to which the states of all agents converge.

Definition 1 (Consensus to subspace S [2]): Let J be an orthonormal matrix in $R^{Nn\times 1}$. The system achieves consensus to the subspace S = span{J}, if S is a minimal set such that for any initial condition the state X(t) converges to a point in J.

In the present work we assume that the desired consensus Sub space S is spanned by the unity vector, i.e. J = **1**.

III.SPACE ANALYSIS

Consider the state space to be composed of two parts, the consensus subspace and its orthonormal subspace.

Assume the orthonormal basis for subspace S is denoted by J_{Nnx1}=1. The orthonormal complement of this matrix is denoted by $J_{\perp}Nnx(Nn-1)$which the basis for the corresponding subspace orthonormal to S.

The following relationships are true for the matrices.

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$$J^*{}_\perp J = 0, \quad J^*{}_\perp J_\perp = I, \quad J^* J = 1, \quad J_\perp J^*{}_\perp + JJ^* = I$$

The state vector can be partitioned into two orthogonal components $X_{s_\perp}$ and X_s according to subspaces $J_\perp$ and J.

$$X = [J_\perp J][X_{s\perp} X_s]^T$$

Assume that the control signal has a state feedback structure, U=KX

Where, K-matrix is feedback gain.
To achieve consensus $X_{J_\perp}$ must be converged to 0.

$$\dot{X}_{s\perp} = J_\perp^*(A + BK)J_\perp X_{s\perp} = A_\perp X_{s\perp} + B_\perp U_\perp$$

Where, $A_\perp = J_\perp^* A J_\perp$, $B_\perp = J_\perp^* B$, $K_\perp = KJ_\perp$, $U_\perp = K_\perp X_{s\perp}$

If this part of the dynamics is stabilized asymptotically to zero, Xs will approach to a constant value which is in the consensus subspace. Therefore, the consensus would be achieved. Now we may design a state feedback control strategy to guarantee the consensus achievement by the closed-loop system.

IV. OPTIMAL CONSENSUS SEEKING

A. Control Design-Purpose

To achieve consensus for the network of multi agent systems the previous three steps were studied in view of the design criteria for stability.

For the subspace $S_\perp = \text{span}\{J_\perp\}$

To achieve this, the goal is to design the control gain $K_\perp$

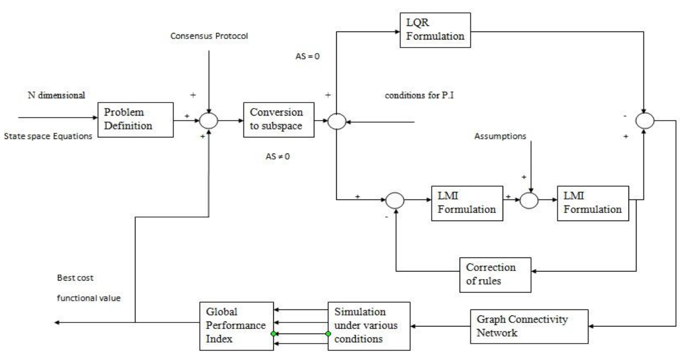

Fig.1. Procedure For consensus

The team conditioning which we use here is a good index of the team performance and its minimization can result in a globally optimal (or suboptimal) solution. However, the solution would be centralized. Fortunately, using the LMI formulation, we will show that this centralized solution can be avoided by adding a constraint on the structure of the controller gain matrix.

B. Discussion on the Solution of Riccati Equation

The optimal control law can be obtained as follows [3]
If the Hamiltonian is defined as

$$H^* = F(X^*, U^*) + \lambda^T G(X^*, U^*)$$

It can be written as

$$H(X,U,\lambda) = ½ [X^T OX + U^T RU] + \lambda [AX+BU]$$

To achieve optimal control,

$$(\delta L/\delta U)_* = 0 \rightarrow (\delta H/\delta U)_* = 0 \text{ as}$$
$$\delta H/\delta U = 0 \rightarrow ½ [2RU^*] + B^T\lambda^* = 0$$
$$RU^* + B^T\lambda^* = 0 \qquad U^* = -R^{-1}B^T\lambda^*$$

Where, $\delta/\delta U \{1/2\ U^T RU\} = RU$ and $\delta/\delta U \{\lambda^* BU\} = B^T\lambda$ are used to determine the state and co-state equations.
From, $(\delta H/\delta U)_* = -\lambda$

$$\text{and} \left(\frac{\delta L}{\delta \lambda}\right)_* = 0 \rightarrow (\delta H/\delta\lambda)_* = \dot{X}^*$$
$$\dot{X}^* = \left(\frac{\delta H}{\delta \lambda}\right)_* \rightarrow \dot{X}^* = AX^* + BU^*$$
$$(d\lambda/dt)^* = -(\delta H/\delta X)_* \rightarrow (d\lambda/dt)^* = -OX^* - A^T\lambda^*$$

Now the boundary condition is clearly variable as $X(t_f)$ is specified to be 1 and X(0) is also specified.

$$\lambda^* = GX^*$$

So, λ^* and X_0 from the two point based boundary value problem.
The closed loop optimal control is obtained under the assumption

$$\lambda^* = PX^* \text{ where P is to be determined.}$$

So, $U^* = -R^{-1}B^T PX^*$ is the negative feedback of X^* .

Differentiate with respect to t

$$d\lambda/dt)_* = \dot{P}X^* + P\dot{X}^*$$
$$\dot{X}^* = AX^* - BR^{-1}BPX^*$$
$$(d\lambda/dt)^* = - OX^* - A^T PX^*$$

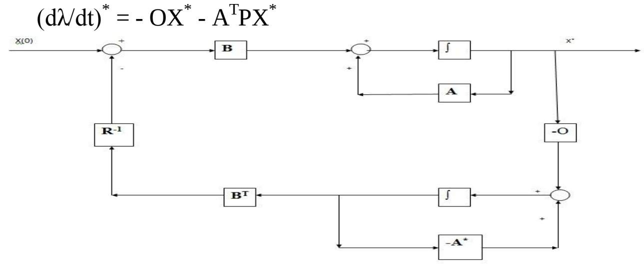

Fig. 2. LQR Sub-system

Now,

$$OX^* - A^T PX^* = \dot{P}X^* + PAX^* - PBR^{-1}B^T PX^*$$
$$(\dot{P} + PA + ATP + O - PBR^{-1}B^T P)X^* = 0$$

As P is independent of X the above equation must hold good for any X^*.

So, the above equation is a matrix differential equation satisfied by P.

It is a differentiable equation of the Riccati time and hence P can be termed as the Riccati Coefficient Matrix.

$$\dot{P} = \quad PA \quad A^T P \quad O + PBR^{-1}B^T P = \quad PA \quad A^T P \quad O + PEP$$

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If $\lambda^* = PX^* = GX^*$ then $P = G$

Optimal value is ½ $X^{T*}PX^*$ and $U^* = -R^{-1}B^TPX^* = -KX^*$

$$K = R^{-1}B^TP \text{ – called Kalman gain – } U = KX$$

For the system to have controllability it is necessary that

$$[B\ AB\ A^2B \_\ \_\ \_\ \_\ \_\ A^{n-1}B]$$

So, $0 = P_\perp A + A^TP_\perp + O - P_\perp BR^{-1}B^TP_\perp$

C. Method for Subspace J

The problem of minimizing the cost function subject to its dynamical conditions is a Linear Quadratic Regulator (LQR) problem.

The solution for such a problem is $U_\perp = -R^{-1}B_\perp^*PX_{S_\perp}$

Where, P must satisfy the Riccati Equation.

$$\hat{O} + P_\perp A + A^TP_\perp - P_\perp BR^{-1}B^TP_\perp = 0$$

So, $K_\perp = KJ_\perp = -R^{-1}BB_\perp^*P$ from properties of the $J_\perp$ matrix, K is determined.

$$K = -R^{-1}B_\perp^*PJ_\perp^*X \text{ for open loop}$$

$$\dot{X} = (A - R^{-1}B_\perp^*PJ_\perp^*)X \text{ for closed loop.}$$

If AJ=0 then, a stable consensus is achieved which is possible if and only if $\det(A^i) = 0$ and matrix A^i is the same for all i.

D. LMI METHOD

The problem of minimizing the cost function cannot be solved by the Linear Quadratic Regulator (LQR) Problem Method. [4]-[6] In order to solve this Optimal Control Problem the Linear Matrix Inequalities (LMI) can be used.

Assume P_1 be a real symmetric solution of the Algebraic Riccati Equation (ARE).

$$A^TP_1 + P_1A - P_1BB^TP_1 + O_1 = 0$$

With $O_1 = O_1^T$ and assume that Re $\{\lambda\,(A\text{-}BB^TP_1)\} < 0$

Then any real symmetric solution P_2 of the ARE

$A^TP_2 + P_2A - P_2BB^TP_2 + O_2 = 0$ with $O_2 = O_2^T$ and $O_1 \geq O_2$ satisfies $(P_1 \geq P_2)$. The main aim is to solve the controller equation $U_\perp = K_\perp X_{S_\perp}$ that minimizes the cost function.

Here, K is the feedback control gain.

For these conditions the Riccati Equation is $\hat{O} + P_\perp A + A^TP_\perp - P_\perp BR^{-1}B^TP_\perp = 0$ .The optimal output for an initial condition X (0) is then given as $X(0)^TPX(0)$. Now in order to further solve the above conditions the concept of convexity must be introduced so as to ensure that the system is feasible.

Consider the condition,

$$X_{S_\perp}X_{S_\perp}^T(0)P^TX_{S_\perp}(0) \text{ Subject to the ARE}$$

$$P(A_\perp + B_\perp K_\perp) + (A_\perp + B_\perp K_\perp)^*P + \hat{O} + K_\perp^*RK_\perp = 0$$

As the ARE must be feasible this further reduces the condition of solution. So,

$$P(A_\perp + B_\perp K_\perp) + (A_\perp + B_\perp K_\perp)^*P + \hat{O} + K_\perp^*RK_\perp \geq 0$$

It must be observed that $X_{S_\perp}^T(0)P^TX_{S_\perp}(0)$ provides a single solution which can be replaced directly by the trace due to the notion that $Y^TLY = y_1^2l_{11} + y_1y_2(l_{12} + l_{21}) + y_2^2l_{22}$

This process is obtained from matrix theory. But for this to be utilized the most important assumption to be considered is that the matrix P is always positive and linearly quadratic.

So, for the dynamic system

$$\dot{X}_{S_\perp} = A_\perp X_{S_\perp} + B_\perp U_\perp$$

The inequality constraints become,

$$P\left(A_\perp + B_\perp K_\perp\right) + (A_\perp + B_\perp K_\perp)^*P + \hat{O} + K_\perp^*RK_\perp \leq 0$$

$P^{-1}J_\perp^*OJ_\perp P^{-1} + P^{-1}J_\perp^*K_\perp^*RK_\perp J_\perp P^{-1} \leq 0$.The LMI for consensus analysis is given by the statement

Theorem: Given a matrix J € $R^{Nn\,xP}$ the autonomous system achieves consensus to S if and only if

(1)AJ = 0 and

(2)There exists X > 0 such that $J_\perp^*(AX + XA^*)J_\perp < 0$

Where X satisfies

$$X = J_\perp J_\perp^*XJ_\perp J_\perp^* + J^*JJX^*$$

Proof – To prove sufficiency,

Assume, AJ = 0 and there exists X>0 such that conditions (1) and (2) are feasible. Now, consider the term $XJ_\perp$, it can be replaced by $J_\perp J_\perp^*XJ_\perp$

Now, $J_\perp^*AXJ_\perp + J_\perp^*XA^TJ_\perp < 0$ and

$$J_\perp^*AXJ_\perp J_\perp^*XJ_\perp + (J_\perp J_\perp^*XJ_\perp)^*A^*J_\perp < 0$$

If $J_\perp^*XJ_\perp$ is replaced with P then $J_\perp^*AJ_\perp P + PJ_\perp^*A^*J_\perp < 0$ which is enough to show that the system achieves consensus. The conditions for consensus are satisfied and there exists a solution P > 0 for the LMI

$$P^TG + GP < 0$$

From the above it is clear the X > 0 and the consensus is achievable.

So, now $XJ_\perp = J_\perp P$ and by the assumption the LMI is feasible.

Now, $J_\perp^*(AX + XA^*)J_\perp < 0$ is achievable as

$$J_\perp^*AJ_\perp P + PJ_\perp^*A^*J_\perp < 0$$

$$J_\perp^*AXJ_\perp + J_\perp^*XA^*J_\perp < 0$$

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$J_\perp^*(AX + XA^*)J_\perp < 0$

Replacing X with a new variable Z

Now, $Z = J_\perp J_\perp^* Z J_\perp J_\perp^* + JJ^* Z JJ^*$

They are positive definite matrices with P begin perfectly invertible i.e. $PP^{-1} = I$

Further, $ZJ_\perp = J_\perp P^{-1}$

Now the inequality can be rewritten as

$$J_\perp^*(AZ + BKZ + ZA^* + ZK^*B^* + ZQZ + ZK^*RKZ)J_\perp \leq 0$$

Replace KZ by W

$J_\perp^*(AZ + BW + ZA^* + ZW^* + ZQZ + W^*RW)J_\perp \leq 0$ To convert this inequality into an LMI the Schur's Complement is used.

E. LMI CONCEPTION BY SCHUR'S COMPLEMENT

Let F: $V \rightarrow S^n$ be an affine function which is partitioned according to

$$F(x) = \begin{bmatrix} F_{11}(x) & F_{12}(x) \\ F_{21}(x) & F_{22}(x) \end{bmatrix}$$

Where $F_{11}(x)$ is square.

Then $F(x) > 0$ if and only if

(1) $F_{11}(x) > 0$
(2) $F_{22}(x) - F_{12}(x) F_{11}^{-1}(x) F_{21}(x) > 0$

Now to apply the Schur's Complement to the problem

$$J_\perp^*(AZ + BW + ZA^* + w^*B^*)J_\perp + J_\perp^*(ZO^{1/2})(O^{1/2}ZJ_\perp) + (J_\perp^* w^* R^{1/2})(R^{1/2}WJ_\perp) \leq 0$$

Assume O > 0 and R > 0 and symmetric then

$$\begin{bmatrix} O & S \\ S^T & R \end{bmatrix} \geq 0$$

is equivalent to $R \geq 0,\ O - SR^tS^T \geq 0,\ S(I - RR^t) = 0$

Where, R^t denotes the Moore Penrose of R.
The Moore Penrose inverse is an approximate inverse of R.
Assume U is an orthogonal matrix that diagonalizes R
Assume,

$$\gamma = J_\perp^*(AZ + BW + w^*B^* + ZA^*)J_\perp$$

Further as $J_\perp^* J_\perp + J_\perp J_\perp^* = 0$

So, if $S_1 = J_\perp^* Z O^{1/2}$ $S_2 = J_\perp^* W^* R^{1/2}$[7]

V. EXISTENCE OF SOLUTIONS

To determine the existence of solutions it is necessary to prove the detectability and stabilizability conditions. In order to explain about the usage of optimality on the system dynamics by LMI methodology consider the following lemma

Lemma 1 - The basic minimization problem or its related LMI's under the application of the dynamic state will have an optimal solution if the matrices A,B and O are given such that satisfy the inequalities for a matrix P_2 .

(1)The stability condition is $J^*(AP_2 + P_2A^* - BB^*) J < 0$
(2)The detect ability condition is $J_\perp^*(P_2A + A^*P_2 - O)J_\perp < 0$
Where $P_2 > 0$ satisfies

$$P_2 = J_\perp J_\perp^* P_2 J_\perp J_\perp^* + JJ^*P_2JJ^*$$

Note – Here P_2 is a new variable with similar properties as that of Z and $P_2 < 0$.

Further if

1. $A_\perp P_1 + P_1 A_\perp^* < 0$ and
2. $P_1 A_\perp + A_\perp^* P_1 < 0$

Then the lemma is satisfied and P_1 is a sub matrix of P_2.

If A, B and O do not satisfy above lemma then the addition of an internal feedback loop to each r is sufficient.

From the above theoretical observations it can be directly inferred that the below conditions are possible in the space J.

The conditions are –

1. If A, B and O are given, then assume U = KX as the control strategy.
Where, $K = WZ^{-1}$ and matrices Ω, W and Z are obtained as follows
Min trace (γ) subject to

a. $\begin{bmatrix} \Omega & I \\ I & J_\perp^* Z J_\perp \end{bmatrix} > 0$

b. $\begin{bmatrix} \gamma & J_\perp^* Z O^{1/2} & J_\perp^* W^* R^{1/2} \\ O^{1/2} Z J_\perp^* & -I & 0 \\ R^{1/2} W J_\perp^* & 0 & -I \end{bmatrix} > 0$

In the process of network design, the simplest tool in mathematics

c. $(AZ + BW)J = 0$
d. $Z = J_\perp J_\perp^* Z J_\perp J_\perp^* + JJ^*ZJJ^*, Z > 0$

2. The controllers are semi decentralized order
   a. Z is a diagonal matrix i.e
   b. $Z = \mathrm{diag}\,[Z_1 \ldots\ldots\ldots\ldots\ldots Z_N]$.
   c. W (i,j) = 0 if L(i,j) = 0.

The above conditions are specified in terms of W and Z.
Neglecting the space conditions
Upon conversion of these terms by applying $Z=P^{-1}$.
As, $K=WZ$
$K=WP^{-1}$

$W=PK$

The conditions of LMI are rewritten as

1. $\Omega\,(P^{-1}) - I^2 > 0$

Post multiply P on both sides

$$P - \Omega < 0$$

2. $\Gamma + ZQZ + W^*RW \leq 0$

$AZ + BW + ZA^* + W^*B^* + ZOZ + W^*RW \leq 0$

$AP^{-1} + BPK + P^{-1}A^* + K^*PB^* + P^{-1}OP^{-1} + K^*B^*RBK \leq 0$

Pre and post multiplying P

$$PA + PBPKP + A^*P + PK^*PB^*P + O + PK^*B^*RBKP \leq 0$$

1. $AZ + BW = 0$
   $K = -AB^{-1}$
2. The fourth condition can be neglected as the space constraints and subspace are not considerable for the above application.

The supporting concepts to achieve these phenomenons are discussed during the application of graph theory to achieve networked consensus.

Further, for the information based matrices all the controllers are dependent on their adjacent controllers to decide on the working and exchange of the network.

A. Connectivity

In the process of network design, the simplest tool in mathematics is graph connectivity. Further, graph connectivity is chosen due to its uniqueness is assigning values or systems and also ease of computation.

The need for usage of Graph theory can be explained by the following Lemma.

Lemma - The closed loop network of multi-agent systems i.e. (A + BK) represents the laplacian matrix of a weighted graph. The corresponding graph is a sub-graph to the original network graph but with different weights assigned to its edges.

Proof – The proof to the above lemma is obtained as the definition of matrices A & B are diagonals and the necessary restrictions on K, the matrix (A + BK) has a similar structures to that of the laplacian network with extra '0' elements.

This matrix will satisfy (A + BK) J = 0.

But it doesn't mean that the solution graph must be connected and hence to show that compulsory connectivity of the network is essential to achieve consensus can be shown using the theorem.

Theorem – If the graph of the entire network of multi–agents is not connected, then existence of a solution for consensus can be guaranteed if and only if (A + BK) represents the laplacian of a connected sub-graph of the original graph.

Proof – The two necessary conditions for consensus are that (A + BK) J = 0 is possible and that K must be designed such that

$\dot{X}_{S\perp} = A_\perp X_{S\perp} + B_\perp U_\perp$ is asymptotically stable.

To ensure asymptotically stability, the matrix $J_\perp^*(A + BK)J$ for $\dot{X}_{S\perp}$ must be Hurwitz i.e. it should have no zero eigen values.

From, $J_{\perp Nnx(Nn-1)}$ it is evident that there are Nn-1 independent column vectors, if they are denoted as

$J_{1\perp}, J_{2\perp} \ldots\ldots J_{(Nn-1)\perp}$

$$J_\perp = [\,J_{1\perp}, J_{2\perp} \ldots\ldots J_{(Nn-1)\perp}\,]$$ [8]

If it is assumed that this network graph is not connected. I t implies that the laplacian matrix L will have more than 1 zero eigen value. Further, (A + BK) also equates the same. (A + BK) has an eigen vector corresponding to the '0' eigen vector which doesn't exist in the S subspace.

VI. RESULTS

The detailed analysis of the presented methods for achieving optimal consensus can be shown using a team of six vehicles. In solving the consensus problem for this particular problem the following statements must be considered.

1. Only distance and velocity are considered as the states and no other external disturbances are considered to affect this network.
2. Only time is considered as the consensus achievement parameter and any such other processes are nonexistent.
3. The network for the application of the consensus protocol is flexible and as such can only be assumed.

These assumptions and/or considerations form the underlying paths for consensus. This is necessitated due to high complexity of the problem even for a two state matrix without any change. The solutions obtained for the procedures are classified under two scenario's i.e. $AJ \neq 0$ and $AJ = 0$. The graph considered to show the network is as follows

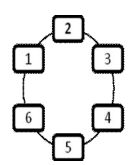
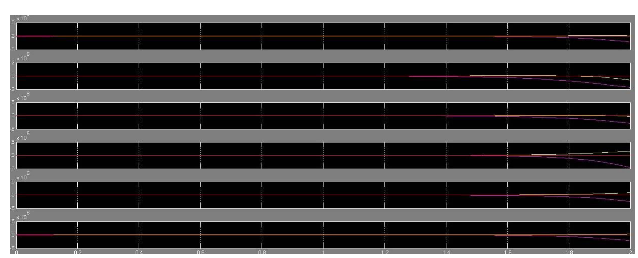


Fig.3. Graphed Network

Fig.4. $AJ \neq 0$ Condition Output



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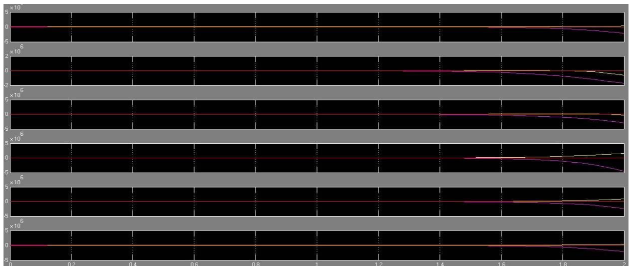

Fig. 5. AJ = 0 Condition Output

The solutions are obtained for a set of values

A. By LQR

B.By LMI

In the process of applying the LMI method to the above network for the same model set of values as provided for the LQR method it must be noted that the network can be decentralized and the final time of network can be considered as the combined sum of the sub systems.

The Robust control toolbox is used in order to solve the network by the LMI method. [9]

The general code for the subsystem is
The following program is used for the AJ = 0 cases:

```
X= [    ]';      % Initial values specifiable for individual subsystem
A= [    ];      % Constant state space array of the subsystems
setlmis([]);
P=lmivar(1,[2 1]);
lmiterm([1 1 1 P],X',X);
lmiterm([2 1 1 P],1,A,'s');
lmiterm([2 1 1 P],.5*1,P*A,'s');
lmiterm([2 1 1 P],.5*A',P,'s');
lmiterm([2 2 1 0],2.4494);
lmiterm([2 2 2 0],-1);
lmiterm([2 3 1 P],0.707,1);
lmiterm([2 3 3 0],-1);
lmis = getlmis
[tmin,xfeas] = feasp(lmis)
P=dec2mat (lmis,xfeas,P)
```

As only time is the parameter of consideration
For AJ $\neq$ 0 the t is 1.5499 and for AJ = 0 the t is 12.00980.

Upon comparing the solutions in particular the time period of staying in consensus of all the subsystems of the network, the following conclusions can be put forward:
1. In the AJ $\neq$ 0 condition the time duration for which the subsystems are in consensus is more for the Riccati Equation Method(LQR) over the LMI method and thus if all the state space's of the subsystems are unequal then the LQR method is best suitable to solve the network.
2. In the AJ = 0 condition the time duration for which the subsystems are in consensus is more for the LMI method over the Riccati Equation Method(LQR) and thus if all the state space's of the subsystems are equal then the LMI method is best suitable to solve the network.

VII. CONCLUSION

An optimal control design strategy is shown in this paper to guarantee consensus achievement in a network of multi-agent systems. It has been shown that the approach based on the Riccati equation, in general, fails to provide a global solution for a stable consensus protocol. Therefore, we have shown an alternative approach for the minimization of a global cost function through a set of constraints that are expressed as LMIs. Contrary to a control methodology that optimizes the individual agent's cost function, by introducing a global cost function, we can ensure only a parameter of consideration neglecting the other considerations. Moreover, through the LMI formulation of the problem, constraints on partial information availability can formally be taken into account. Therefore, corresponding to the individual agent control design, the only imposed requirement is that the information should be made available and received from the neighbours in an agent's neighbouring set (defined as the agents that are connected to this agent in the network underlying graph). Finally, it should be noted that our proposed framework has sufficient flexibility for accommodating additional constraints and design criteria in the development of consensus seeking protocols.

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