ISSN 2320 – 2602
Volume 3, No.10, October 2014
International Journal of Advances in Computer Science and Technology
Available Online at http://warse.org/pdfs/2014/ijacst013102014.pdf

ANALYSIS OF PERFORMANCE MEASURES OF FUZZY QUEUING MODEL WITH AN UNRELIABLE SERVER USING RANKING FUNCTION METHOD

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ABSTRACT

In this paper we propose a procedure to find the various performance measures in terms of crisp values for fuzzy queuing model with an unreliable server where the arrival rate, service rate, breakdown rate and repair rate are all fuzzy numbers. Here the inter arrival time, service time, breakdown and repair rates are Triangular and also Trapezoidal fuzzy numbers. Our idea is to convert the fuzzy inter arrival rate, service rate breakdown rate and repair rate into crisp values by applying ranking function method. Then apply the crisp values in the classical queuing performance measure formulas. Ranking fuzzy numbers plays a huge role in decision making under fuzzy environment. This ranking method is most reliable method, simple to apply and can be used for all types of queuing problems. A numerical example is solved successfully for both triangular and trapezoidal fuzzy numbers.

Key words : Fuzzy sets, Fuzzy queues, Fuzzy ranking, Membership Functions, Unreliable server.

1 . INTRODUCTION

In most queuing models, it is not possible to keep the server operational at all times, and service can thus be interrupted. The main reason for this is the breakdown of the server. There can also be scheduled service interruptions such as during weekends or holidays. Regarding queuing model with server breakdown, Gaver [1] first proposed an ordinary M/G/I queuing system with interrupted service and priorities. His system was extended to the GI/G/I case by Sengupta [2]. Li et al. [3] and Wang [4] investigated the behavior of the unreliable server, and the effect the server breakdowns and repairs in the M/G/I queuing models and he investigated the controllable M/HK/I queuing systems with an unreliable server recently. Gurukajan and Srinivasan [5] described a complex two-unit system in which the repair facility is subject to random breakdown.

Through Zadeh's[6] extension principle, we transform the fuzzy queue with an unreliable server into a family of crisp queues with an unreliable server. For notational convenience our model in this paper will be denoted by FM/FM(FM,FM)/I where the first to the fourth FM represents the fuzzified exponential arrival, service, breakdown and repair rate respectively. In contrast M/M(M,M)/I model represents the exponential inter-arrival (Poisson), service, breakdown and repair rates respectively.

The method for ranking was first proposed by Jain [7]. Yager [8] proposed four indices which may be employed for the purpose of ordering fuzzy quantities in [0,1]. In Kaufmann and Gupta [9], an approach is presented for the ranking of fuzzy numbers. Campos and Gonzalez [10] proposed a subjective approach for ranking fuzzy numbers. Liou and Wang [11] developed a ranking method based on integral value index. Cheng [12] presented a method for ranking fuzzy numbers by using the distance method. Kwang and Lee [13] considered the overall possibility distributions of fuzzy numbers in their evaluations and proposed a ranking method. Modarres and Nezhad [14] proposed a ranking method based on preference function which measures the fuzzy numbers point by point and at each point the most preferred number is identified. Chu and Tsao [15] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu [16] presented a centroid-index method for ranking fuzzy numbers. Liang et al. [17] and Wang and Lee [18] also used the centroid concept in developing their ranking index. Chen and Chen [19] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [20] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α-levels of trapezoidal fuzzy numbers. Ranking methods also have been analyzed by such researchers like F.Choobinesh and H.Li[21], S.H.Chen[22], R.Nagarajan andA.Solairaju[23].

In this paper we develop a method that is able to provide performance measures in terms of crisp values for fuzzy queuing model with an unreliable server with four fuzzy variables, namely fuzzified exponential arrival rate, service rate, breakdown rate and repair rate. Here ranking function method has been used to reach crisp values.

2 . PRELIMINARIES

2.1 Definition

A fuzzy set is determined by a membership function mapping elements of a domain space or universe of discourse Z to the unit interval [0,1].

(i,e) $\tilde{A}$ = {(z, $\mu_{\tilde{A}}$(z)); z Є Z}.

Here $\mu_{\tilde{A}}$: Z →[0,1] is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}$(z)is called the membership value of z Є Z in the fuzzy set $\tilde{A}$. These membership grades are often represented by real numbers ranging from [0,1].

2.2 Definition (normal)

A fuzzy set $\tilde{A}$ of the universe of discourse Z is called a normal fuzzy set if there exists at least one z Є Z such that $\mu_{\tilde{A}}$(z)= 1.

2.3 Definition (convex)

A fuzzy set $\tilde{A}$ is convex if and only if for any ,z Є Z, the membership function of $\tilde{A}$ satisfies the condition

$\mu_{\tilde{A}}$ {λ z_1 +(1-λ) z_2 } ≥ min { $\mu_{\tilde{A}}$(z_1), $\mu_{\tilde{A}}$(z_2)}, 0 ≤ λ ≤ 1.

2.4 Definition (Triangular fuzzy number)

A triangular fuzzy number $\tilde{A}$(z) can be represented by $\tilde{A}(a_1, a_2, a_3; 1)$ with membership function $\mu_{\tilde{A}}$(z) given by

$$\mu_{\tilde{A}}(z) = \begin{cases} \dfrac{z-a_1}{a_2-a_1}, & a_1 \le z \le a_2 \\ 1 \quad , & z = a_2 \\ \dfrac{z-a_3}{a_2-a_3}, & a_2 \le z \le a_3 \\ 0, otherwise \end{cases}$$

2.5 Definition (Trapezoidal fuzzy number)

A trapezoidal fuzzy number $\tilde{A}$(z) can be represented by $\tilde{A}(a_1, a_2, a_3, a_4; 1)$ with membership function $\mu_{\tilde{A}}$(z) given by

$$\mu_{\tilde{A}}(z) = \begin{cases} \dfrac{z-a_1}{a_2-a_1}, & a_1 \le z \le a_2 \\ 1 \quad , & a_2 \le z \le a_3 \\ \dfrac{z-a_4}{a_3-a_4}, & a_3 \le z \le a_4 \\ 0, otherwise \end{cases}$$

2.6 Definition (Generalized fuzzy number)

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a generalized fuzzy number if its membership function has the following characteristics:

1) $\mu_{\tilde{A}}$: R → [0,w] is continuous;

2) $\mu_{\tilde{A}}$ (z) = 0 for all z $\in (-\infty, a_1] \cup [a_4, \infty)$;

3) $\mu_{\tilde{A}}$ (z) strictly increasing on [a_1, a_2] and strictly decreasing on [a_3, a_4];

4) $\mu_{\tilde{A}}$ (z) = w, for all z ∈ [a_2, a_3], where 0 < w ≤ 1.

2.7 Definition (L-R type generalized fuzzy number)

A Fuzzy number $\tilde{A}$ = $(a_1, a_2, a_3, a_4; w)_{LR}$ is said to be an L-R type generalized fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(z) = \begin{cases} wL\left(\dfrac{a_2-z}{a_2-a_1}\right), & a_1 \le z \le a_2 \\ w \quad , & a_2 \le z \le a_3 \\ wR\left(\dfrac{z-a_3}{a_4-a_3}\right), & a_3 \le z \le a_4 \\ 0, otherwise \end{cases}$$

where L and R are reference functions.

2.8 Definition (Generalized Triangular fuzzy number)

A generalized Triangular fuzzy number $\tilde{A}$(z) can be represented by $\tilde{A}(a_1, a_2, a_3; w)$ with membership function $\mu_{\tilde{A}}$(z) given by

$$\mu_{\tilde{A}}(z) = \begin{cases} \dfrac{w(z-a_1)}{(a_2-a_1)}, & a_1 \le z \le a_2 \\ w \quad , & z = a_2 \\ \dfrac{w(z-a_3)}{(a_2-a_3)}, & a_2 \le z \le a_3 \\ 0, otherwise \end{cases}$$

2.9 Definition (Generalized Trapezoidal fuzzy number)

A generalized trapezoidal fuzzy number $\tilde{A}$(z) can be represented by $\tilde{A}(a_1, a_2, a_3, a_4; 1)$ with membership function $\mu_{\tilde{A}}$ (z) given by

$$\mu_{\tilde{A}}(z) = \begin{cases} \dfrac{w(z-a_1)}{(a_2-a_1)}, & a_1 \le z \le a_2 \\ w \quad , & a_2 \le z \le a_3 \\ \dfrac{w(z-a_4)}{(a_3-a_4)}, & a_3 \le z \le a_4 \\ 0, otherwise \end{cases}$$

3 . FUZZY QUEUING MODEL WITH AN UNRELIABLE SERVER

Consider a fuzzy queuing system with an unreliable server and two different types of breakdowns. In type I, the server may breakdown even if there are no customers in the system and in type II, the server may breakdown when there is at least one customer in the system. It is assumed that the customers arrive at a single server facility as a Poisson process with fuzzy rate $\tilde{\lambda}$, the service times as an exponential distribution with fuzzy rate $\tilde{\mu}$, the server may have a breakdown following Poisson process with fuzzy rate $\tilde{\alpha}$, and the repair follows an exponential distribution with fuzzy rate $\tilde{\beta}$ are approximately known and can be represented by convex fuzzy sets. Let $\phi_{\tilde{\lambda}}(x)$, $\phi_{\tilde{\mu}}(y)$, $\phi_{\tilde{\alpha}}(s)$ and $\phi_{\tilde{\beta}}(t)$ denote the membership functions of $\tilde{\lambda}$, $\tilde{\mu}$, $\tilde{\alpha}$ and $\tilde{\beta}$. Then we have the following fuzzy sets:

$$\tilde{\lambda} = \left\{ (x, \phi_{\tilde{\lambda}}(x)) \,\middle|\, x \in X \right\}$$

$$\tilde{\mu} = \left\{ (y, \phi_{\tilde{\mu}}(y)) \,\middle|\, y \in Y \right\}$$

$$\tilde{\alpha} = \left\{ (s, \phi_{\tilde{\alpha}}(s)) \,\middle|\, s \in S \right\}$$

$$\tilde{\beta} = \left\{ (t, \phi_{\tilde{\beta}}(t)) \,\middle|\, t \in T \right\}$$

Where X,Y,S and T are the crisp universal sets of the arrival , service, breakdown and repair rates respectively. Let f(x,y,s,t) denote the system characteristic of interest. Since x, y, s and t are fuzzy numbers. f(x,y,s,t) is also a fuzzy number.. Let A and B represents the membership function of the expected time that the system is idle in type I and type II respectively.

$$A = f(x,y,s,t) = \frac{ty - x(s+t)}{y(s+t)} \quad (1)$$

$$B = f(x,y,s,t) = \frac{ty - x(s+t)}{ty} \quad (2)$$

In steady-state, it is required that $0 < \dfrac{ty - x(s+t)}{y(s+t)} < 1$ and $0 < \dfrac{ty - x(s+t)}{ty} < 1$.

4 . RANKING FUNCTION METHOD – ALGORITHM

To solve the problem we defuzzify the fuzzy numbers into crisp ones by a fuzzy number ranking function R : F(R) → R where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists.

(i) . Let a convex triangular fuzzy number

$\tilde{A}$(z) = $\tilde{A}(a_1, a_2, a_3; w)$,

Then the Ranking Index is defined by

$$R(\tilde{A}) = \int_0^w \frac{(L^{-1}(z) + R^{-1}(z))}{2} dz$$

Where $L^{-1}(z) = a_1 + (\dfrac{a_2 - a_1}{w})z$ and

$$R^{-1}(z) = a_2 + (\frac{a_3 - a_2}{w})z$$

$$\Rightarrow \quad R(\tilde{A}) = \frac{w(a_1 + 2a_2 + a_3)}{4} \quad (3)$$

(ii) . Let a convex trapezoidal fuzzy number

$\tilde{A}$(z) = $\tilde{A}(a_1, a_2, a_3, a_4; w)$,

Then the Ranking Index is defined by

$$R(\tilde{A}) = \int_0^w \frac{(L^{-1}(z) + R^{-1}(z))}{2} dz$$

Where $L^{-1}(z) = a_1 + (\frac{a_2 - a_1}{w})z$ and

$$R^{-1}(z) = a_3 + (\frac{a_4 - a_3}{w})z$$

$$\Rightarrow \quad R(\tilde{A}) = \frac{w(a_1 + a_2 + a_3 + a_4)}{4} \qquad (4)$$

5 . NUMERICAL EXAMPLE

Consider a railway computer reservation system in which people using single-channel reservation arrivals according to a Poisson process. The time required to reservation process may be interrupted (computer, server, or printer problems) following a Poisson process. The recovery times of the problem interrupted follows an exponential distribution. The reservation resumes as soon as the interruption ends. In attempting to evaluate the single-channel reservation, the manager of the system wishes to evaluate how many hours that the system is idle. It is evident that this system follows, FM/FM(FM,FM)/1/∞ and the expected time that the system is idle can be derived by the proposed procedure.

5.1 For Triangular fuzzy number

Suppose the arrival rate, service rate, breakdown rate and repair rates are triangular fuzzy numbers represented by
$\tilde{\lambda}$ = [2,4,7;1], $\tilde{\mu}$ = [12,14,20;1], $\tilde{\alpha}$ = [0.05, 0.1, 0.5;1],
$\tilde{\beta}$ = [2,3,6;1] per hour.

According to (3)
The ranking index of $\tilde{\lambda}$ is

$$R(\tilde{\lambda}) = R(2,4,7;1) = \frac{(2+8+7)}{4} = 4.25$$

Proceeding similarly we get

$$R(\tilde{\mu}) = R(12,14,20;1) = \frac{(12+28+20)}{4} = 15$$

$$R(\tilde{\alpha}) = R(0.05, 0.1, 0.5;1) = \frac{(0.05+0.2+0.5)}{4} = 0.19$$

$$R(\tilde{\beta}) = R(2,3,6;1) = \frac{(2+6+6)}{4} = 3.5$$

According to (1) , (2)
The expected time that the system is idle in

$$\text{Type I} = \frac{ty - x(s+t)}{y(s+t)}$$

$$= \frac{3.5(15) - 4.25(0.19+3.5)}{15(0.19+3.5)} = 0.66$$

$$\text{Type II} = \frac{ty - x(s+t)}{ty}$$

$$= \frac{3.5(15) - 4.25(0.19+3.5)}{3.5(15)} = 0.7$$

5.2 For Trapezoidal fuzzy number

Suppose the arrival rate, service rate, breakdown rate and repair rates are trapezoidal fuzzy numbers represented by
$\tilde{\lambda}$ = [2,4,5,7;1], $\tilde{\mu}$ = [12,14,16,20;1],
$\tilde{\alpha}$ = [0.05, 0.1, 0.2, 0.5;1], $\tilde{\beta}$ = [2,3,5,6;1] per hour.
According to (4)
The ranking index of $\tilde{\lambda}$ is

$$R(\tilde{\lambda}) = R(2,4,5,7;1) = \frac{(2+4+5+7)}{4} = 4.5$$

Proceeding similarly we get

$$R(\tilde{\mu}) = R(12,14,16,20;1) = \frac{(12+14+16+20)}{4} = 15.5$$

$$R(\tilde{\alpha}) = R(0.05, 0.1, 0.2, 0.5;1) = \frac{(0.05+0.1+0.2+0.5)}{4} = 0.2125$$

$$R(\tilde{\beta}) = R(2,3,5,6;1) = \frac{(2+3+5+6)}{4} = 4$$

According to (1) , (2)
The expected time that the system is idle in

$$\text{Type I} = \frac{ty - x(s+t)}{y(s+t)}$$

$$= \frac{4(15.5) - 4.5(0.2125+4)}{15.5(0.2125+4)} = 0.659$$

$$\text{Type II} = \frac{ty - x(s+t)}{ty}$$

$$= \frac{4(15.5) - 4.5(0.2125+4)}{4(15.5)} = 0.694$$

6 . CONCLUSION

In this paper, Fuzzy set theory has been applied to a queuing model with an unreliable server. Fuzzy queuing model with an unreliable server models have been used in operations and service mechanism for evaluating system performance. Moreover, the fuzzy problem has been transformed into crisp problem using ranking indices. Since the performance measures such as the expected idle time for two types are crisp values, the manager can take the best and optimum decisions.

We conclude that the solution of fuzzy problems can be obtained by Ranking function method very effectively. The approach proposed in this paper provides practical information for system manager and practitioners.

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